Pictures of Calculus: Learning That Lasts

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Learning calculus, retaining that learning, and being able to transfer that learning to different situations (applications) is our goal for all our calculus students. One of the three principles derived from the results of learning research over recent years, as reported in the book *How People Learn* is:

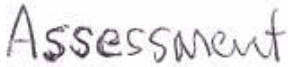
To develop competence in an area of inquiry, students must (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of the factual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

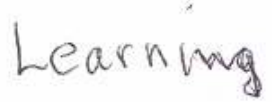
From my own learning experiences, especially from my own study of calculus and other mathematics, I have found that pictures (graphs, diagrams, etc.) help me to understand facts and conceptual structures for those facts and to recall and use those facts and concepts.

Another connection of pictures to developing competence concerns a major difference between novices and experts. Experts seem to organize their knowledge and recall that knowledge in batches, while novices' knowledge is often organized in small bits and recalled accordingly. Pictures are ways of batching knowledge.

This presentation will focus on how I believe pictures help to organize and explain calculus.

What follows are several somewhat cryptic sketches of ideas of calculus. You will likely recognize some of them and the ideas that they represent. In this session, I will fill in the details and remove any mysteries of the pictures (with transparency overlays) and tell you why I find the pictures so helpful in organizing, illuminating, and recalling information.





Assessment of Learning

What is it?
and
Why is it important ?

The Mathematical Association of America

Most college faculty believed that assessment was, as the name implied, only some kind of comprehensive evaluation.

They knew, as did every farmer, that weighing one's produce did not hasten its readiness for market.

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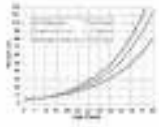
Unless one keeps records and observes what treatments — food, environment, etc — increase the profits from the produce, and responds accordingly.



What is assessment?

Comparing student learning with the learning goals of an academic program or curricular block of an academic program.

Why is assessment done?

- The University administration mandated it.
- The governing board mandated it.
- The legislature mandated it.
- The accrediting agency mandated it.
- To evaluate academic programs.
- To evaluate students.
- To evaluate faculty performance.

What are better reasons for assessment?

- To improve courses.
- To improve academic programs.
- To improve teaching.
- To enhance student learning.

Principles of Assessment

- Assessment is a not a single event but a continuous cycle.
- Assessment must be an open process.
- Assessment must promote valid inferences.
- Assessment that matters should always employ multiple measures of performance.
- Assessment should measure what is worth learning, not just what is easy to measure.
- Assessment should support every student's opportunity to learn important mathematics.

The Assessment Cycle

Answers three questions:

- What should our students learn?
- How well are they learning?
- What should we change so that future students will learn more and understand it better?

Assessment Cycle in More Detail

1. Articulate the learning goals of the curricular block and a set of objectives that should lead to the accomplishment of those goals.

2. Design strategies (e.g. curriculum and instructional methods) that will accomplish the objectives, taking into account student learning experiences and diverse learning styles, as well as research on how students learn.

Assessment Cycle in More Detail

3. Determine the areas of student activities and accomplishments in which quality will be judged. Select assessment methods designed to measure student progress toward completion of goals and objectives.

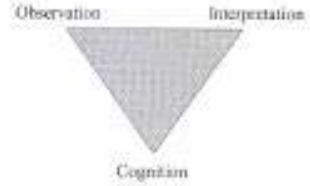
4. Gather assessment data; summarize and interpret the results.

Assessment Cycle in More Detail

5. Use the results of the assessment to improve the curricular block --- the payoff.

6. Return to 1.

ASSESSMENTS MUST REST ON THREE PILLARS...



Assessment Cycle -- Briefer

- Set learning goals and objectives.
- Design strategies to accomplish objectives.
- Determine areas and methods of assessment.
- Gather assessment data.
- Use the assessment data to improve program.
- Do it again.

Goals 1-3 for AP Calculus*

- Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.
- Students should understand the meaning of the definite integral both as a limit of Riemann sums and as a net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.

*[illegible]

Goals 4-6 for AP Calculus

- Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.
- Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

Goals 7-9 for AP Calculus

- Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.
- Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Assessment & Calculus

- Be sure of what you are testing for.
  - Look at goals and objectives to help decide.
  - Sometimes items are so broad one cannot determine misunderstandings.

- Determine what students do to demonstrate learning/understanding. (Notice that AP calculus learning goals are silent on this.)

- Learn from your successes and failures.

- Give students feedback – in class and on written exams and homework.

- Make students aware of kinds of assessment – formative & summative – and purposes of assessment and how the results will be used.

- Make students aware of the nature of the AP calculus examinations.

FUNCTIONS	Rate of Change	NUMBER SENSE	TECHNOLOGY	CONNECTIONS	COMMUNICATION
	Accumulation				
	Approximation				
	Applications				

Other possibilities: Representations
Differential Equations

1997 AB EXAM

At what point on the graph of $y=\frac{1}{2}x^2$ is the tangent line parallel to the line $2x-4y=3$?

$(A)\left(\frac{1}{2},-\frac{1}{2}\right)$

$(B)\left(\frac{1}{2},\frac{1}{8}\right)$

$(C)\left(1,-\frac{1}{4}\right)$

$(D)\left(1,\frac{1}{2}\right)$

$(E)(2,,2)$

Notes: See where the distractors come from. Two of the points are not on the graph of $y=\frac{1}{2}x^2$.

The area of the region enclosed by the graph of $y=x^2+1$ and the line $y=5$ is

$(A)\frac{14}{3}$

$(B)\frac{16}{3}$

$(C)\frac{28}{3}$

$(D)\frac{32}{3}$

$(E)8\pi$

Look at the regions that have areas equal to the distractors.

$\lim_{x\to 1}\frac{x}{\ln x}$ **is**

(A) 0
(B) 1/e
(C) 1
(D) e
(E) nonexistent

Why would a student choose 1/e?

1995 AB-1

Let f be the function given by $f(x) = \dfrac{2x}{\sqrt{x^2 + x + 1}}$.

(a) Find the domain of *f*. Justify your answer.
(b) In the viewing window provided below, sketch the graph of *f*.
(c) Write an equation for each horizontal asymptote for the graph of *f*.
(d) Find the range of *f*. Use $f'(x)$ **to justify your answer.**

Note: $f'(x) = \dfrac{x+2}{(x^2+x+1)^{\frac{3}{2}}}$

2000 AB-5/BC-5

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.
(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.
(c) Find the x-coordinate of each point on the curve where the tangent is vertical.

Levels the technology playing field.
Focuses the constructs being tested for on graphical analysis and optimization.

Lessons Learned 1990-2000

Good examination questions that make essential use of technology are difficult to write. Examination questions that neutralize technology are easier to write.

What you value in students' responses will change from computation and manipulation to analysis, setups, interpretations, and justifications.

Students' responses will be more difficult to evaluate.

Difficult questions about forms of answers will arise.