

The Goat/Cow Grazing in the Grass/Seaweed Problem

I. Find the parametric equations of the involute of the circle.

$$x = a \cos t + at \sin t$$

$$y = a \sin t - at \cos t$$

II. Find the length of the boundary of the grazing region.

$$2 \left[\int_0^{\pi} \sqrt{(at \cos t)^2 + (at \sin t)^2} dt + \frac{2\pi(\pi a)}{4} \right] = 2\pi^2 a$$

III. Find the area of the grazing region.

$$2 \left[- \int_0^{\pi} (a \sin t - at \cos t) at \cos t dt + \frac{\pi(\pi a)^2}{4} \right] - \pi a^2 = \frac{5\pi^3 a^2}{6}$$

IV. Try to find the surface area and the volume of the grazing region of the sea cow.

Surface Area:

$$2\pi \int_0^{\pi} (a \sin t - at \cos t) at dt + \frac{4\pi(\pi a)^2}{2} = 2\pi^2(\pi + 3)a^2$$

Volume:

$$-\pi \int_0^{\pi} (a \sin t - at \cos t)^2 (at \cos t) dt + \frac{4\pi(\pi a)^3}{6} = \frac{\pi(2\pi^3 + 9\pi^2 - 32)a^3}{3}$$

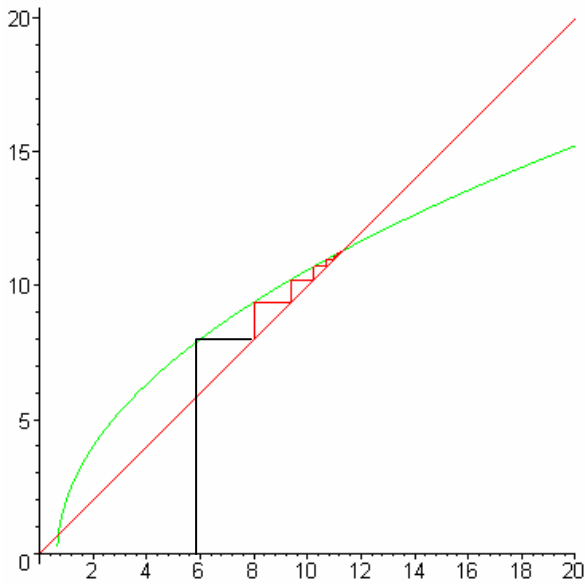
V. Attempt all the previous calculations: Length, area, surface area, and volume.

Enjoy!!!

Iteration and More Grazing

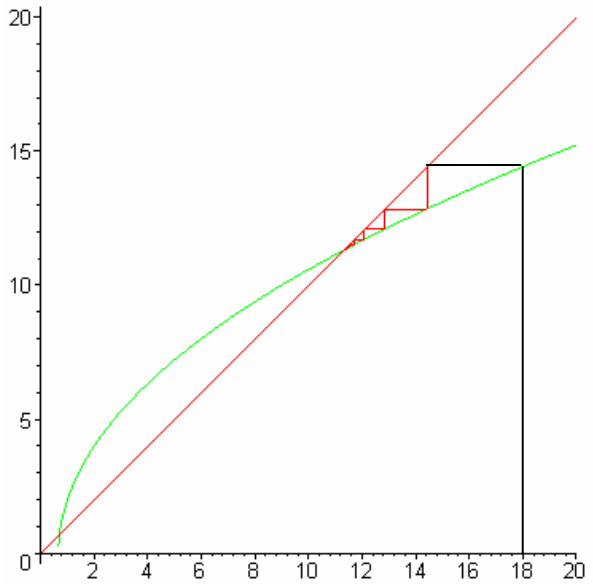
I. Analyze the following recursively defined sequences using a cobweb diagram:

a) $a_1 = 6, a_{n+1} = 2\sqrt{3a_n - 2}$



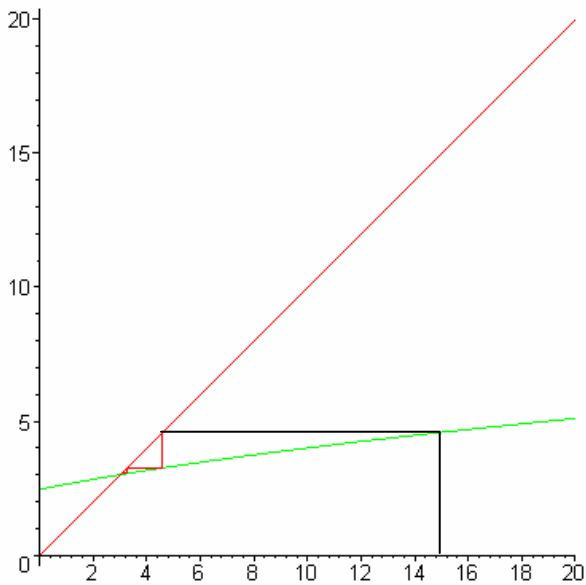
$$\lim_{n \rightarrow \infty} a_n = 6 + 2\sqrt{7}$$

b) $a_1 = 18, a_{n+1} = 2\sqrt{3a_n - 2}$



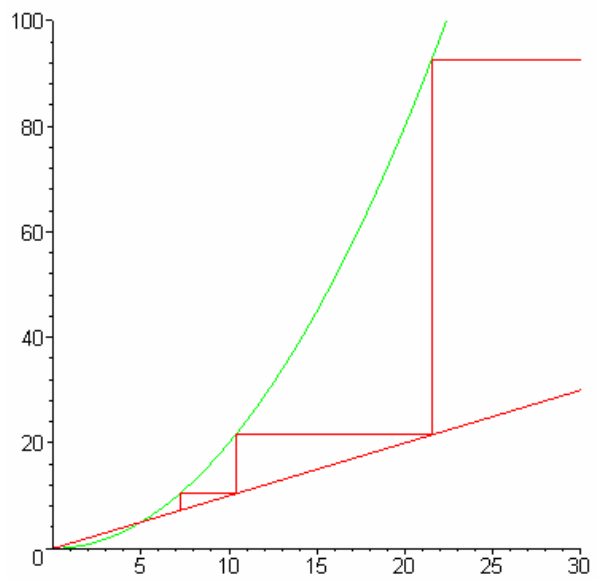
$$\lim_{n \rightarrow \infty} a_n = 6 + 2\sqrt{7}$$

c) $a_1 = 15, a_{n+1} = \sqrt{6 + a_n}$



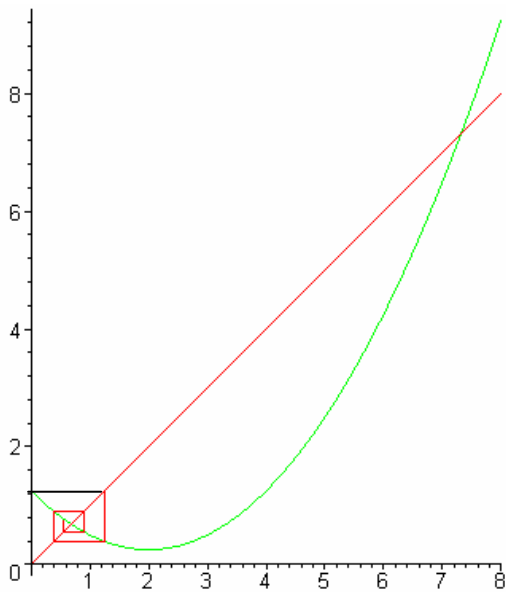
$$\lim_{n \rightarrow \infty} a_n = 3$$

d) $a_1 = 6, a_{n+1} = \frac{1}{5}a_n^2$



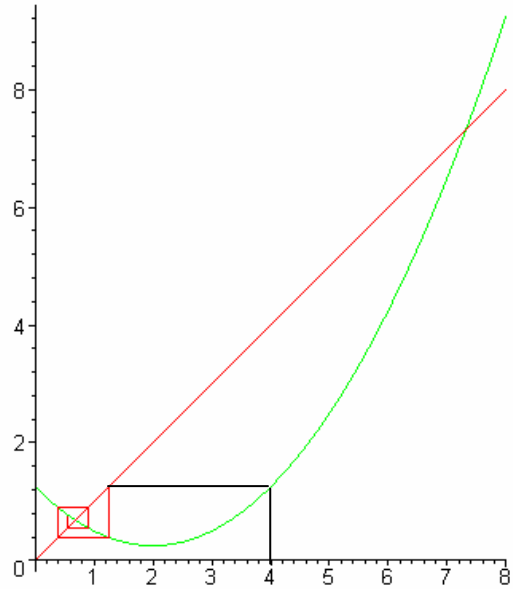
$$\lim_{n \rightarrow \infty} a_n = \infty$$

e) $a_1 = 0, a_{n+1} = \frac{(a_n - 2)^2 + 1}{4}$



$\lim_{n \rightarrow \infty} a_n = 4 - \sqrt{11}$

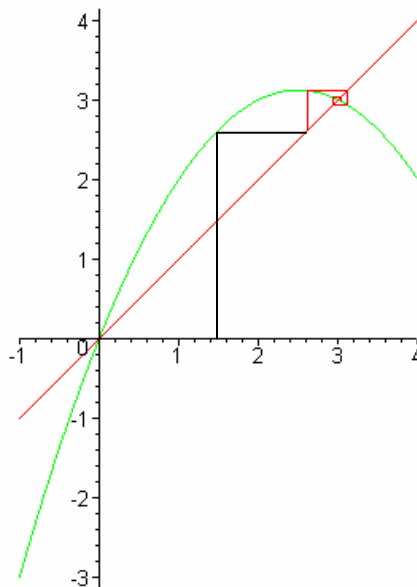
f) $a_1 = 4, a_{n+1} = \frac{(a_n - 2)^2 + 1}{4}$



$\lim_{n \rightarrow \infty} a_n = 4 - \sqrt{11}$

g) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} a_n$ whose terms are defined

recursively by the following: $a_1 = \frac{3}{2}; a_{n+1} = \frac{a_n(5 - a_n)}{2}$



$\lim_{n \rightarrow \infty} a_n = 3$, so the series diverges by n^{th} term test.