Problem Solving with the TI-89/92

Selwyn Hollis

Armstrong Atlantic State University

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left(f(a) + 2\sum_{k=1}^{n-1} f(a+k\,h) + f(b)\right) \text{ where } h = \frac{b-a}{n}$$

.5(b–a)/n*(f(a) + 2*Σ(f(a+k*(b–a)/n), k, 1, n–1) + f(b)) → trapez(a, b, n)

Better: .5(b–a)/n*((fn|x=a) + 2*Σ(fn|x=a+k*(b–a)/n, k, 1, n–1) + (fn|x=b)) → trapez(fn, x, a, b, n)

Newton's Method

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

(1) x – f(x)/*d*(f(x), x) → newtstep(x)

(2) *d*(f(x), x) → df(x) : x – f(x)/df(x) → newtstep(x)

(3) 10^(–8) → δ : x1 – (fn|x=x1)*2δ/((fn|x=x1+δ) – (fn|x=x1–δ)) → newtstep(fn, x, x1)

The Secant Method

$$\frac{1}{f'(x_n)} \approx \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\,f(x_n)}{f(x_n) - f(x_{n-1})} = \frac{x_n(f(x_n) - f(x_{n-1})) - (x_n - x_{n-1})\,f(x_n)}{f(x_n) - f(x_{n-1})} = \frac{x_{n-1}\,f(x_n) - x_n\,f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

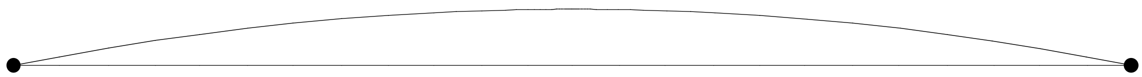
$$x_{n+1} = \frac{x_{n-1}\,f(x_n) - x_n\,f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

{ xpair[2], (xpair[1] (fn|x=xpair[2]) – xpair[2] (fn|x=xpair[1]))/((fn|x=xpair[2]) – (fn|x=xpair[1])) } → secstep(fn, x, xpair)

expression → f(x)
{ x_0, x_1 }
secstep(f(x), x, ans(1))
secstep(f(x), x, ans(1))
⋮

A Surveying Problem

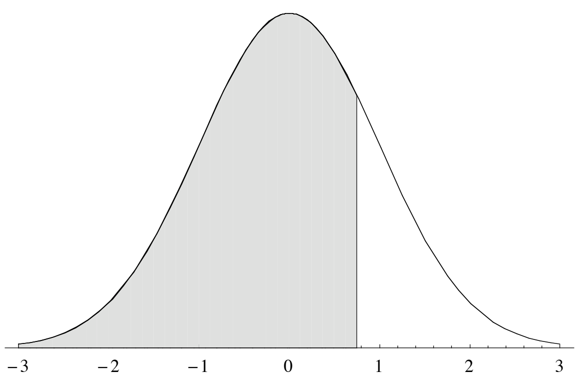
The length of a circular arc between points P and Q is 40 meters. The straight-line distance between P and Q is 36 meters. Find the radius of the circular arc.



Percentiles of the Normal Distribution

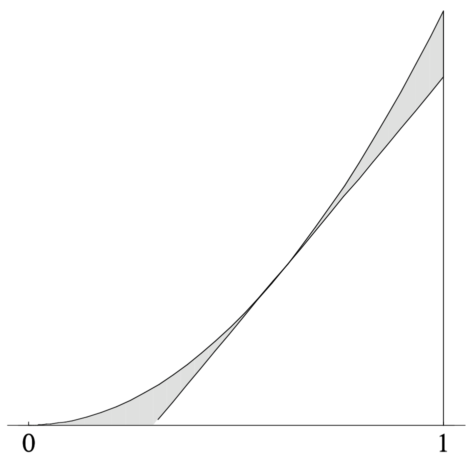
$$P(x < a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-x^2/2} \, dx$$

Find a such that $P(x < a) = 3/4$.

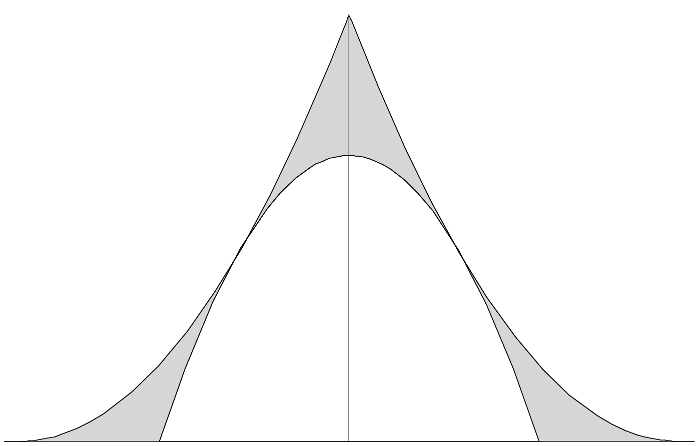


A Pair of Max-min Problems

(1) Let $\mathcal{R}$ be the region bounded by $y = x^2$, $x = 1$, $y = 0$, and a tangent line to $y = x^2$, as shown in the figure. Find the tangent line that minimizes the area of $\mathcal{R}$.



(2) The shaded region in the figure is bounded by $y = (1 - |x|)^2$, $y = b - a x^2$, and $y = 0$.



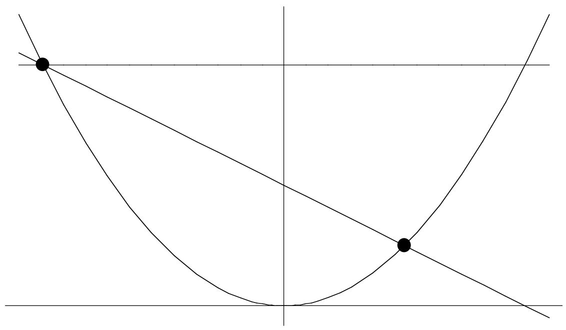
(a) If $a = 2$, find the value of b that minimizes the area of the shaded region.

(b) If $b = 1/2$, find the value of a that minimizes the area of the shaded region.

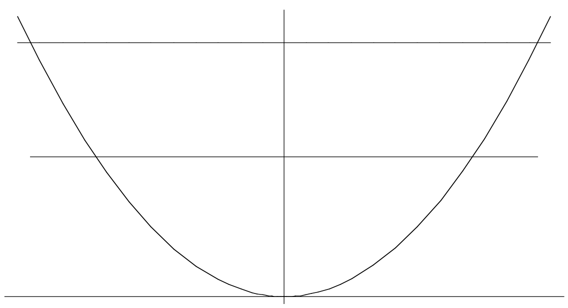
(c) Find the *pair* of values (a, b) that minimizes the area of the shaded region. (Partial derivatives required.)

Some Area Problems

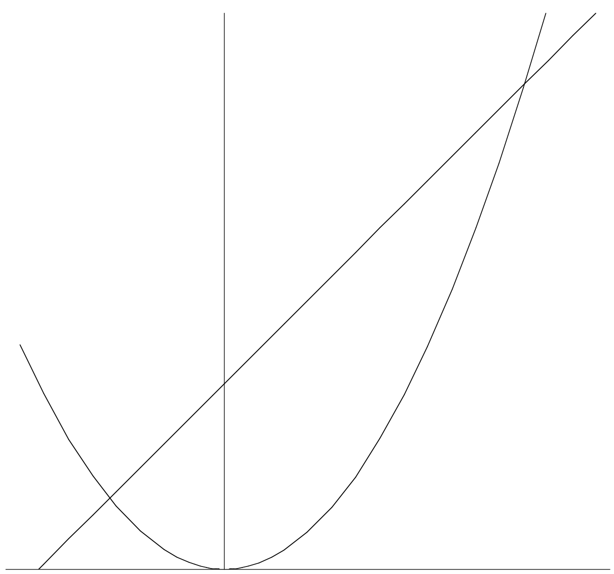
(1) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = 1$. Find the line through $(-1,\ 1)$ that divides $\mathcal{R}$ into two pieces of equal area.



(2) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = 1$. Find the horizontal line $y = b$ that divides $\mathcal{R}$ into two pieces of equal area.



(3) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = x + b$, where $b \geq 0$. Find b so that the area of $\mathcal{R}$ is 4.

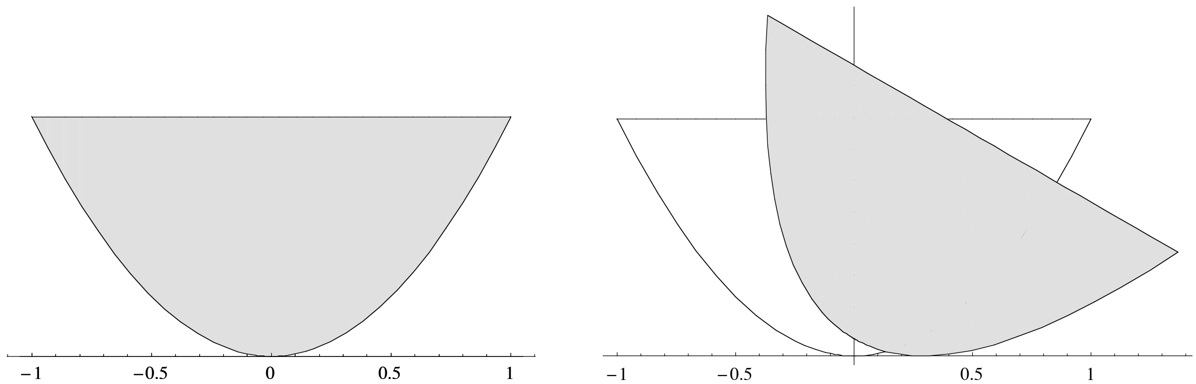


(4) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = mx + 1$, where $m \geq 0$. Find m so that the area of $\mathcal{R}$ is 4.

(5) Let $\mathcal{R}$ be the region bounded by $y = x^2$ and $y = mx + b$, where $m,\ b \geq 0$. Describe all pairs $(m,\ b)$ for which the area of $\mathcal{R}$ is 4.

A Problem About Centroids

(1) A thick plate, with uniform thickness and mass density, has the shape of the region bounded by the parabola $y = x^2$ and the line $y = 1$. If the plate is balanced on its vertex, in what position will it come to rest if it is nudged very slightly to the right?



(2) What if the plate has the shape of the region bounded by the parabola $y = x^2$ and the line $y = 3/4$?

(3) Suppose that the shape of the region bounded by the parabola $y = x^2$ and the line $y = b$. Find the greatest value of b for which the plate will not roll to a balancing point other than the vertex of the parabola.

(4) Consider again the plate with the shape of the region bounded by the parabola $y = x^2$ and the line $y = 1$. Is it possible to drill out a circular hole, centered on the y-axis, so that the upright position becomes stable? If so, what is the minimum radius of the hole?

Tethered to a Parabola

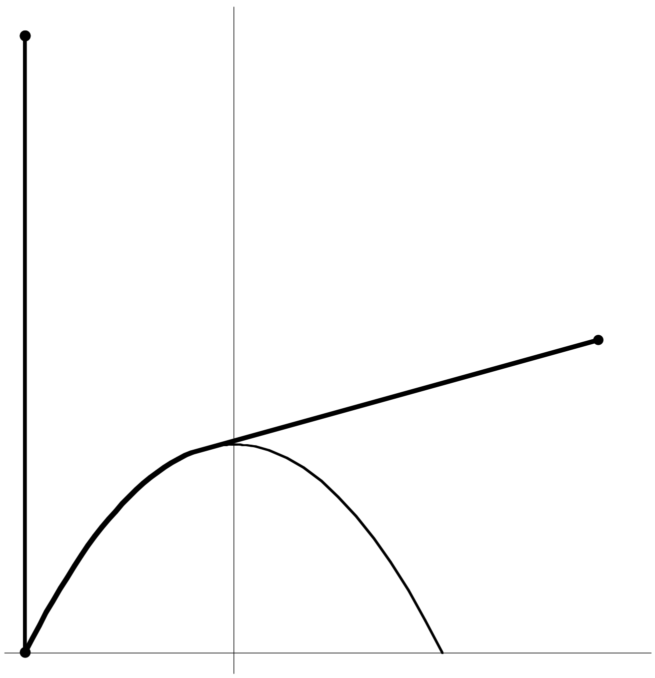
This is a variation on the following sort of problem:

> A barn is 20 ft by 40 ft. Outside the barn, a 50 ft rope tethers a goat to a point at the middle of one of the barn's longer sides. Find the area of the region in which the goat can graze.

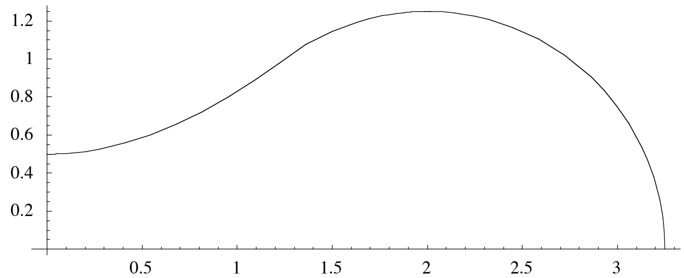
In that problem, if the goat wanders while holding the rope tight, he will trace out a path consisting of five circular arcs.

■ Problem:

One end of a string is fixed at $(-1, 0)$, and the string is initially pulled tight with its other end at the point $(-1, L)$, where L is the length of the string. The top end of the string is then pulled to the right and down, always pulling the string taut and wrapping it over the parabola $y = 1 - x^2$, finally ending up at $(1, 0)$. Find the curve traced out by the end of the string.



The Light-bulb Curve



Let $f(x) = \begin{cases} a x^3 + b x^2 + c x + 1/2 & \text{when } 0 \le x < 1.25 \\ \sqrt{1.25^2 - (x-2)^2} & \text{when } 1.25 \le x \le 3.25 \end{cases}$

(1) Find a, b, c.

(2) Find the volume of bulb.

(3) Find the surface area of bulb.