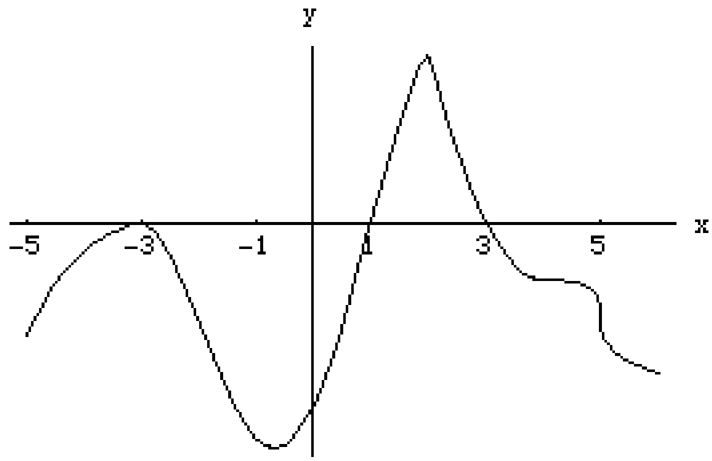
1. Find A and B so that $f(x) = Axe^{Bx}$ has a local minimum of 6 when $x = 2$.

2. The graph below is the graph of f', the derivative of f; The domain of the derivative is $-5 \le x \le 6$. Note there is a cusp when x=2, a horizontal tangent when x=4, and a vertical tangent when x=5



1) The critical points for f are $x =$ ____________________

2) The critical points for f' are $x =$ ____________________

3) f has a local maximum when $x =$ ____________________

4) f has its maximum value on $[-5, 6]$ when $x =$ ____________________

5) f is decreasing on the interval(s) ____________________

6) The graph of f is concave up on the interval(s) ____________________

7) The x-coordinates of the points of inflection are $x =$ ____________________

8) f' has its maximum value when $x =$ ____________________

9) f'' has its maximum value when $x =$ ____________________

10) Does f' have a minimum value on $[-5, 6]$? Explain. ____________________

11) Does f'' have a minimum value on $[-5, 6]$? Explain. ____________________

3. Numeric Stem

Given that f, f', and f'' are all continuous for all x, use the information in the table to answer the questions that follow.

x	$f(x)$	$f'(x)$	$f''(x)$
2	6	2	–8
4	12	0	–1
6	15	3	0
8	20	4	5
10	25	2	6

Fill in the blanks with TRUE, FALSE, or NOT DETERMINED

1) f has a local minimum at $x = 8$ ______________________________

2) f has a local maximum at $x = 4$ ______________________________

3) f has a POI when $x = 6$ ______________________________

4) f has a POI on the interval $6 < x < 10$ ______________________________

5) f is increasing on $[2,10]$ ______________________________

6) $f(x) = 17$ has a solution in $[2,10]$ ______________________________

7) $f'(x) = 2.25$ has a solution in $[6,8]$ ______________________________

8) $f'(x) = 2.50$ has a solution in $[6,8]$ ______________________________

9) $f'(x) = 2.75$ has a solution in $[6,8]$ ______________________________

10) The line $y = 15$ is a horizontal asymptote. ______________________________

11) The line $x = 7$ is a vertical asymptote. ______________________________

4. Analytic Stem

Find all values of A so that $y = \sin(Ax)$ is a solution to $\frac{d^2y}{dx^2} + 9y = 0$, A constant.

5. Numeric Stem

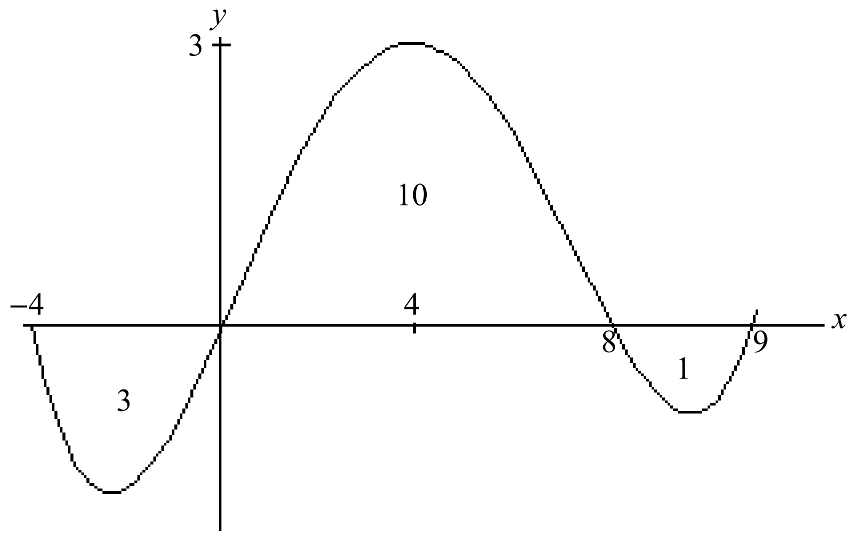
The table gives values for the speed and gas mileage of a car at 10 minute intervals.

t	0	10	20	30	minutes
$v(t)$	40	66	60	54	miles per hour
$m(t)$	26	22	24	25	miles per gallon

1) Write a left-hand Riemann sum to approximate the distance traveled in this 30 minute interval.

2) Write a left-hand Riemann sum to approximate the total fuel consumption in this 30 minute interval.

6. Geometric Stem



The numbers given above in each region give the area of that region. Compute the following numbers.

1) The average value of f on $[0,8]$. ____________________

2) $\int_{-4}^{0} f(x)\,dx$ ____________________

3) $\int_{0}^{9} f(x)\,dx$ ____________________

4) $\int_{0}^{9} f'(x)\,dx$ ____________________

5) $\int_{0}^{4} f'(x)\,dx$ ____________________

6) If $g' = f$, and $g(0) = 6$, what is $g(8)$? ____________________

7) If $g' = f$, and $g(0) = 6$, what is $g(-4)$? ____________________

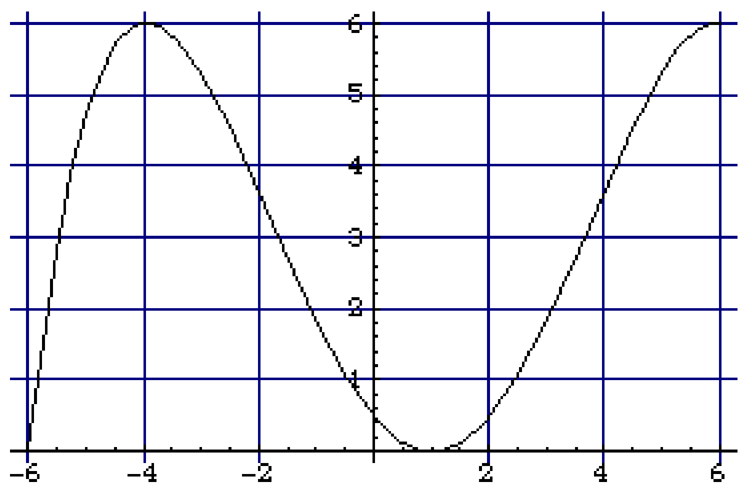
8) If $g' = f$, and $g(0) = 6$, what are the maximum and minimum values of g on $[-4, 9]$? ____________________

7. Analytic Stem (or is it numeric?) Calculator needed.

If $f'(x) = \sin(x^2)$ and $f(0) = 1$, what is $f(2)$?

8. Geometric Stem. The graph of $y = f(x)$ is shown below.

This grid is:
$-6 \le x \le 6$ and $0 \le y \le 6$



1) Estimate $f'(0)$

2) Estimate $\int_0^4 f(x)\,dx$

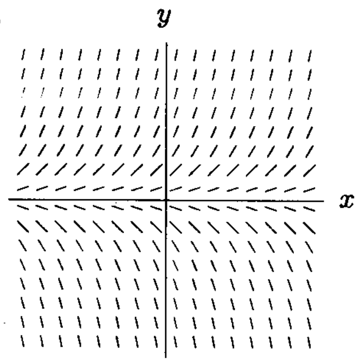
9. Match the slope fields with their differential equations. *Calculus, Hughes-Hallett et.al.*

(a) $y' = 1 + y^2$ (b) $y' = x$ (c) $y' = \sin x$

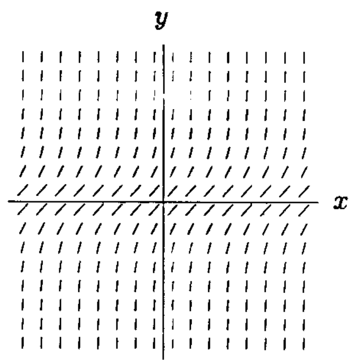
(d) $y' = y$ (e) $y' = x - y$ (f) $y' = 4 - y$

Each slope field is sketched for $-5 \le x \le 5,\ -5 \le y \le 5$

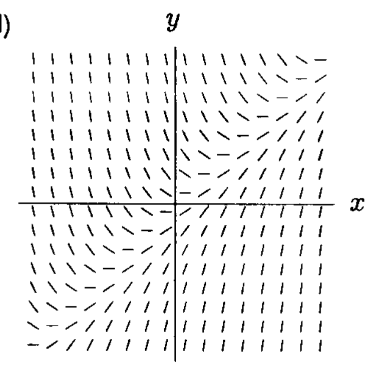
(I)



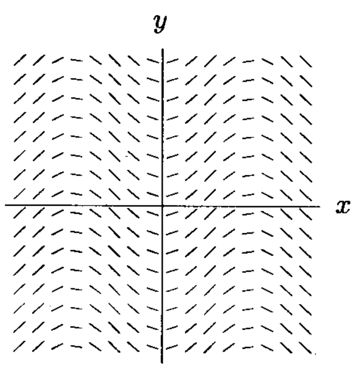
(II)



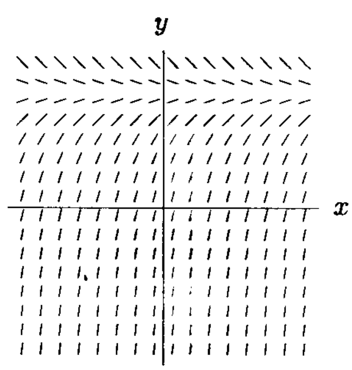
(III)



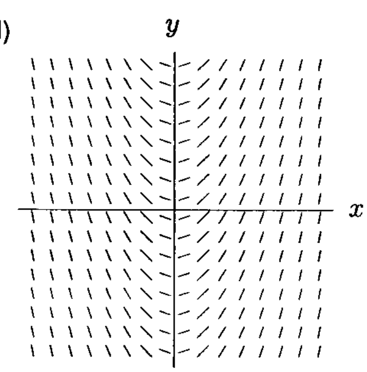
(IV)



(V)



(VI)



10. Given the function $F(x) = \int_2^x \frac{dt}{t - t^2}$.

(a) $F'(x) =$

(b) $\lim_{x \to 2} F(x)$

(c) Use Trapezoidal Rule with $n = 4$ to approximate $F(4)$.

(d) What is the largest meaningful domain for F?

11. A hemispherical tank of radius 4 feet contains water to a depth of 2 feet. The widest part of the tank is at the top.

(a) How much work is done in pumping the water to a point 3 feet above the tank?

(b) How much water must be added to the tank to raise the level of water by 1 foot?

12. $$\lim_{n\to\infty}\left[\frac{1}{n\sqrt{1+\left(\frac{1}{n}\right)^2}}+\frac{1}{n\sqrt{1+\left(\frac{2}{n}\right)^2}}+\cdots+\frac{1}{n\sqrt{1+\left(\frac{n}{n}\right)^2}}\right]=$$

13. $$\lim_{x\to 2^+}\frac{[x]-2}{x-2}$$

14. Assume that f and g are differentiable functions defined on all of the real line. Mark each of the following TRUE or FALSE.

(a) It is possible that $f > 0, f' > 0,$ and $f'' < 0$ everywhere.

(b) f can satisfy: $f'' > 0, f' < 0,$ and $f > 0$ everywhere.

(c) f and g can satisfy: $f'(x) > g'(x)$ and $f(x) < g(x)$ for all x.

(d) If $f'(x) = g'(x)$ for all x and if $f(x_0) = g(x_0)$ for some x_0, then $f(x) = g(x)$ for all x.

(e) If $f''(x) < 0$ and $f'(x) < 0$ everywhere then $\lim_{x\to\infty} f(x) = -\infty$

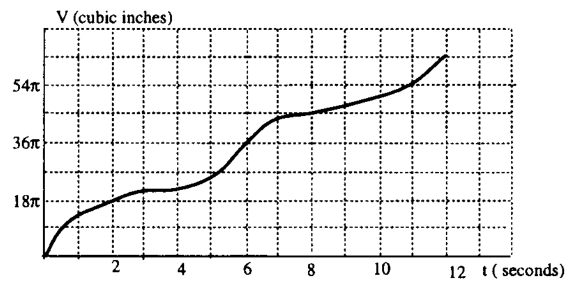
15. The acceleration due to gravity, g, is given by $g = \frac{GM}{r^2}$, where M is the mass of the earth, r is the distance from the center of the earth, and G is the universal gravitational constant.

(a) Show that when r changes by Δr, the change in the acceleration due to gravity, Δg is given by $\Delta g \approx -2g\frac{\Delta r}{r}$.

(b) What is the significance of the negative sign?

(c) What is the percent change in g when moving from sea level to the top of Pike's Peak (4.315 km)? Assume the radius of the earth is 6400 km.
Hughes-Hallett 3rd ed.

16. The function V whose graph is sketched below gives the volume of air, $V(t)$, that a man has blown into a balloon after t seconds. Assuming the balloon maintains a spherical shape as it expands, approximately how rapidly is the radius changing after 6 seconds? $\left(V = \frac{4}{3}\pi r^3\right)$



17. Let $g'(x) = x^4 \sin(2x) - e^{(x^2)}$. What value of x produces the absolute minimum for $g(x)$ on the interval $[-2\pi, 2\pi]$?

18. Sketch the graph of a function $f(x)$ that has all of the given properties. The function f has exactly one discontinuity and one stationary point.

$$\lim_{x\to\infty} f(x) = 1, \quad \lim_{x\to-\infty} f(x) = -1, \quad \lim_{x\to 3^-} f(x) = \infty, \quad \lim_{x\to 3^+} f(x) = -\infty, f(2) = 0, \text{ and } f'(2) = 0.$$

19. Let f be differentiable for all real numbers. Which of the following **must** be in the range of f?

a. $f(a) - f(b)$ b. $f(a) + f(b)$ c. $\frac{f(a) + f(b)}{2}$ d. $\frac{f(a) - f(b)}{2}$ e. None of these

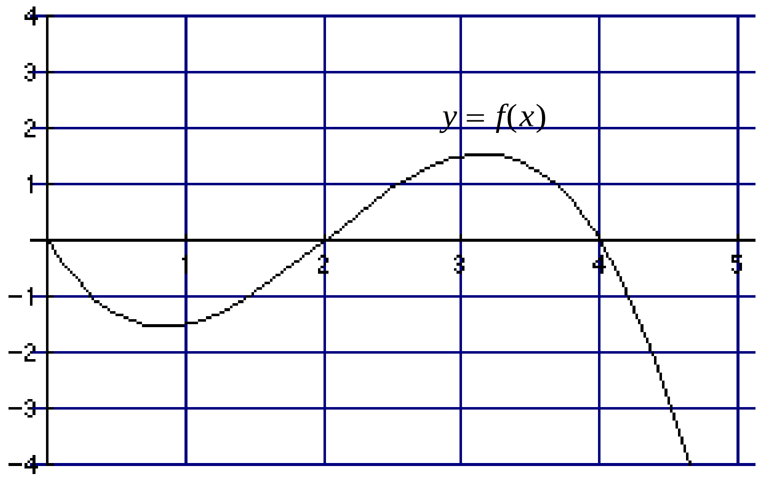
20. (a) $\lim_{n\to\infty}\left[\frac{1}{n^2\sqrt{1+\left(\frac{1}{n}\right)^2}} + \frac{2}{n^2\sqrt{1+\left(\frac{2}{n}\right)^2}} + \cdots + \frac{n}{n^2\sqrt{1+\left(\frac{n}{n}\right)^2}}\right] =$

(b) $\lim_{n\to\infty}\left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{n+n}\right]$

(c) $\lim_{h\to 0}\frac{1}{h}\int_x^{x+h} \frac{1}{u+\sqrt{u^2+1}}\,du$

(d) $\lim_{x\to x_1} \frac{x}{x - x_1}\int_{x_1}^{x} f(t)\,dt$

21. $G(x)=\int_1^x f(t)\,dt$, use the graph of $y=f(x)$ to answer the following questions.

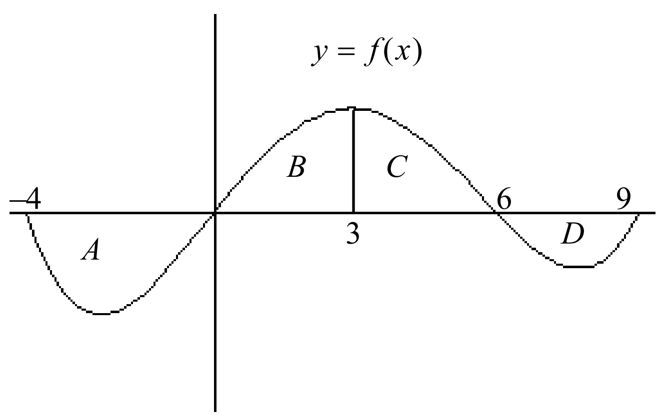


(a) (2 pts.) Estimate the slope of the curve $y=G(x)$ at $x=2$ and $x=1$.

(b) (2 pts.) Estimate $G(0)$

(c) (2 pts.) Estimate $G(3.01)-G(3)$

(d) (4 pts.) Sketch the graph of $y=G(x)$.



22. Let h be the function defined by $h(x)=\int_0^{x^2} f(t)\,dt$ where the graph of f is shown above. The regions labeled A, B, C, and D have the following areas: $A=B=4$, $C=3$, and $D=2$

(a) Is h is an even function, an odd function, or neither? Justify your answer.
(b) Find the domain of h.
(c) Determine all solutions to $h'(x)=0$.
(d) Sketch the graph of $y=h(x)$ over its entire domain.

23. Given the differential equation $\frac{dy}{dx} = \frac{-xy}{\ln y}$, $y > 0$.

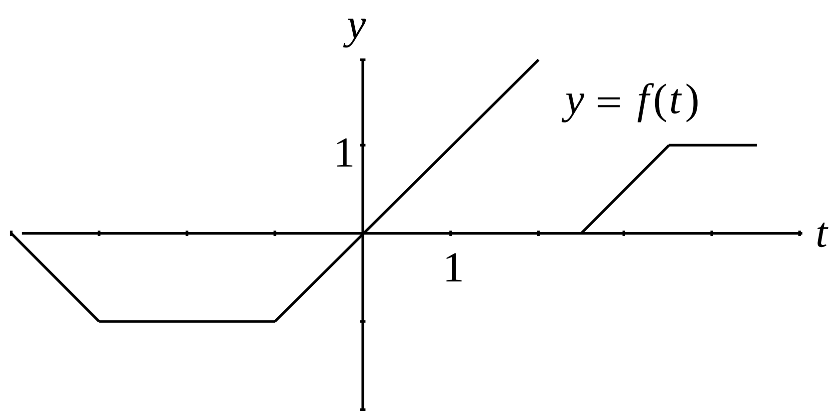
(a) Find the general solution of the differential equation.
(b) Find the particular solution for $y(0) = e^{-2}$. Express answer in the form $y = f(x)$.
(c) Explain why $x = 2$ is not in the domain of the solution found in part (b).

24. Consider the differential equation $\frac{dy}{dx} = 2x + y$.

(a) Verify that the general solution is $y = ke^{x} - 2x - 2$.
(b) Find the particular solution for $y(0) = 1$.

25. Variables x and y are related by the equation $x = \int_0^y \frac{1}{\sqrt{1+4t^2}}\, dt$.

Show that $\frac{d^2y}{dx^2}$ is proportional to y and determine the constant of proportionality.



Domain of f = $\{-4 \le t \le 2\} \cup \{2.5 \le t \le 4.5\}$

26. Let $g(x) = \int_{-1}^{2x} f(t)\, dt$ and $h(x) = \int_{4}^{2x} f(t)\, dt$.

(a) What is the domain of h ?
(b) What is the domain of g ?
(c) For what values of x does $g'(x) = -\frac{1}{2}$?
(d) Sketch the graph of $y = g(x)$ over its entire domain.

27. A particle is moving on the x-axis, where x is in centimeters. Its velocity, v, in cm/sec, when it is at the point with coordinate x is given by $v = x^2 + 3x - 2$. Find the acceleration of the particle when it is at the point $x = 2$. Give the units of your answer. *Hughes-Hallett et. al. 3rd ed.*

Answers

1. No such values of A and B.
2. 1) -3, 1, 3
   2) -3, -1, 2, 4, 5
   3) 3
   4) -5
   5) [-5, 1], [3, 6]
   6) (-5, -3), (-1, 2)
   7) -3, -1, 2
   8) 2
   9) 1
   10) Yes, since f' is continuous the Extreme Value Theorem applies, f' attains both a maximum and a minimum value on [-5, 6].
   11) No, f'' does not exist at $x = 5$ and $\lim_{x \to 5} f(x) = -\infty$

3. 1) False
   2) True
   3) Not Determined
   4) True
   5) False
   6) True
   7) Not Determined
   8) True
   9) True
   10) Not Determined
   11) False

4. $A = -3, 0, 3$

5. a) $\frac{10}{60}(40 + 66 + 60)$ miles

   b) $\frac{10}{60}\left(\frac{40}{26} + \frac{66}{22} + \frac{60}{24}\right)$ gallons

6.

1) 5/4	5) 3
2) -3	6) 16
3) 9	7) 9
4) 0	8) Max = 16, Min = 6

7. $f(x)=1+\int_0^x \sin(t^2)\,dt,\ \ f(2)\approx 1.805$

8. 1) $f'(0)\approx -1$ 2) $\int_0^4 f(x)\,dx \approx 4.5$

9. a) II b) VI c) IV d) I e) III f) V

10. a) $\dfrac{1}{x-x^2}$ b) 0 c) -0.4196 d) $1<x<\infty$

11. a) 14,898.689 ft.-lbs. b) $\dfrac{41}{3}\pi$ cubic feet

12. AB: 0.881, BC: $\ln\left(1+\sqrt{2}\right)$ Both

13. 0

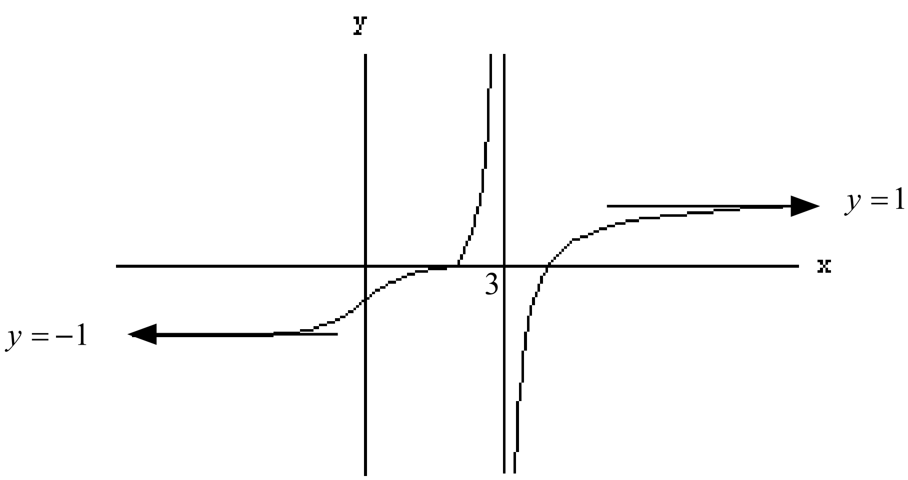
14. a) F b) T c) T d) T e) T

15. b) The acceleration due to gravity decreases as the distance from the center of the earth increases.

c) -0.135%

16. $\dfrac{1}{3}\ \dfrac{\text{in}}{\text{sec}}$ 17. 2π

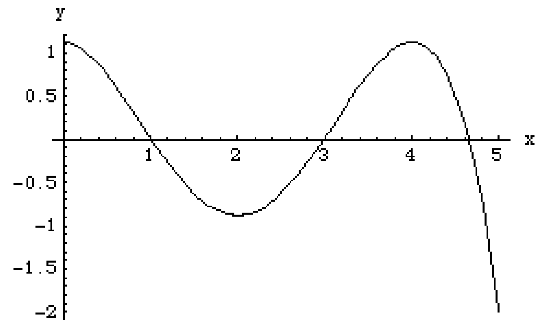
18.



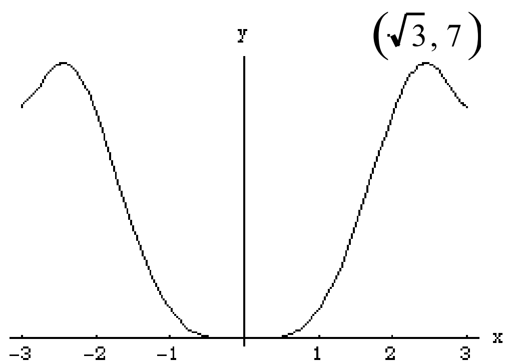
19. c

20. a. $\sqrt{2}-1 \approx 0.414$ b. $\ln 2$ c. $\dfrac{1}{x+\sqrt{x^2+1}}$ d. $x_1 \cdot f(x_1)$

21. a. $0, -1.5$ b. anything near 1.125 c. 0.015 d. below



22. a. even b. $-3 \le x \le 3$ c. $0, \pm\sqrt{6}, \pm 3$ d. below

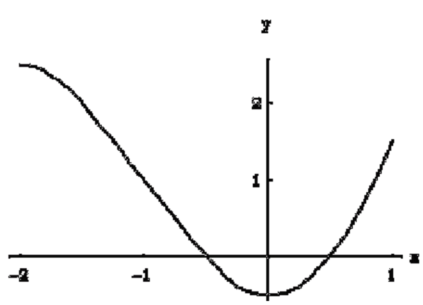


23. a. $y = e^{\pm\sqrt{C-x^2}}$ b. $y = e^{-\sqrt{4-x^2}}$

c. $x = 2$ would make $y(2) = 1$ and the denominator of $\dfrac{dy}{dx}$ would be zero.

24. b. $y = 3e^x - 2x - 2$ 25. Constant = 4

26. a. $1.25 \le x \le 2.25$ b. $-2 \le x \le 1$ c. $x = -\dfrac{1}{8}, -\dfrac{15}{8}$ d. below



27. 56 $\text{cm}/\text{sec}/\text{sec}$