name: ______________________________
date: ______________________________

AP Calculus AB/BC
Optimization Match Up

1.

Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

Main equation you're optimizing:

Helper equation:

Optimizing equation after substitution (1 variable only):

Good sketch of optimizing equation graph (provide scale):

2.

A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with the largest volume?

Main equation you're optimizing:

Helper equation:

Optimizing equation after substitution (1 variable only):

Good sketch of optimizing equation graph (provide scale):

3.

An open rectangular box with square base is to be made from 12 ft^2 of material. What dimensions will result in a box with the largest possible volume?

Main equation you're optimizing:

Helper equation:

Optimizing equation after substitution (1 variable only):

Good sketch of optimizing equation graph (provide scale):

LABEL AXES w/ PROBLEM TERMINOLOGY (NOT x & y)

4.

Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

Main equation you're optimizing:

Helper equation:

Optimizing equation after substitution (1 variable only):

Good sketch of optimizing equation graph (provide scale):

5.

A container in the shape of a right circular cylinder with no top has surface area 12π ft^2. What height h and base radius r will maximize the volume of the cylinder?

Main equation you're optimizing:

Helper equation:

Optimizing equation after substitution (1 variable only):

Good sketch of optimizing equation graph (provide scale):

6.

A cylindrical can is to hold 12π m^3. The material for the top and bottom costs \$12/m^2, and material for the side costs \$8/m^2. Find the radius r and height h of the most economical can.

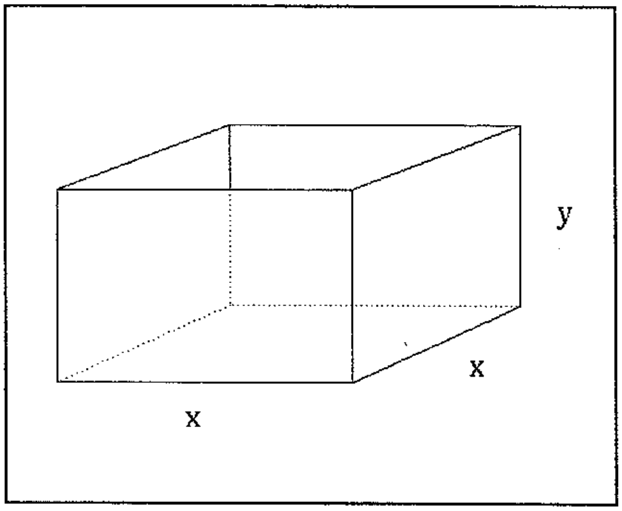
Main equation you're optimizing:

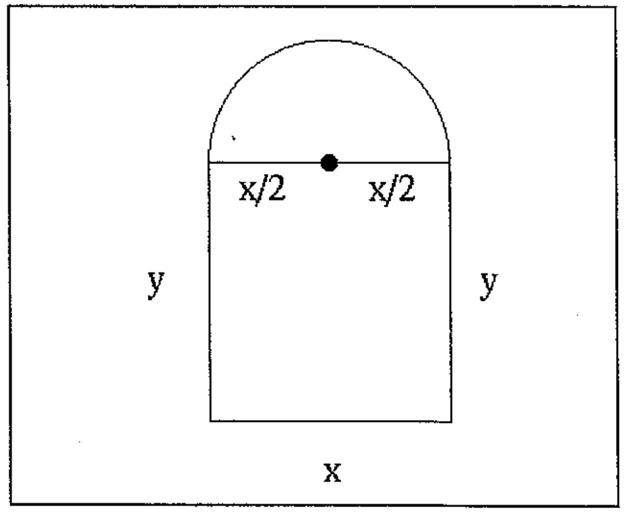
Helper equation:

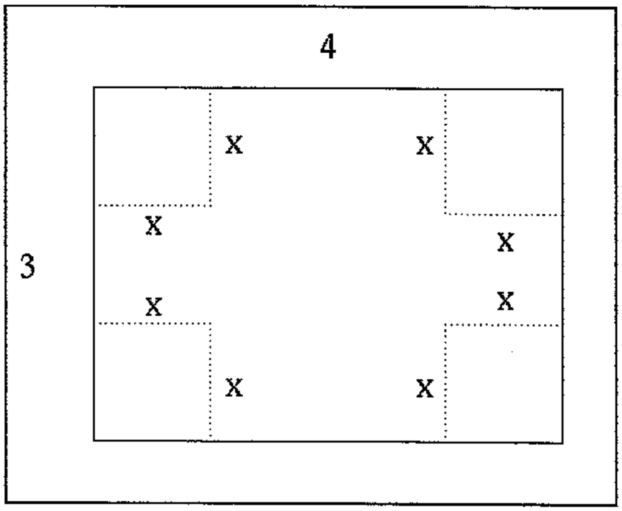
Optimizing equation after substitution (1 variable only):

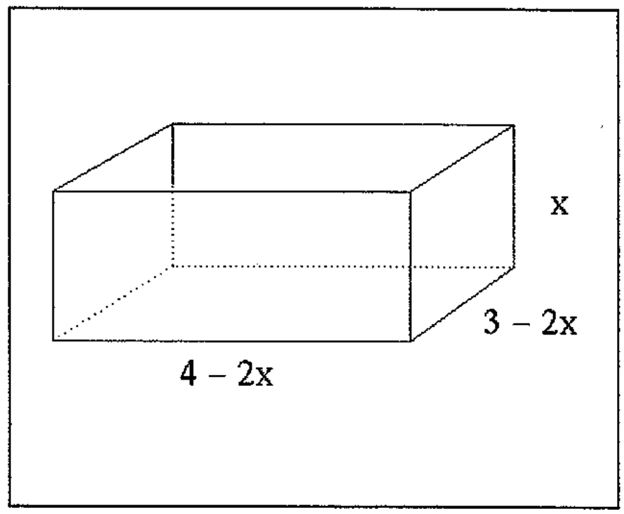
Good sketch of optimizing equation graph (provide scale):

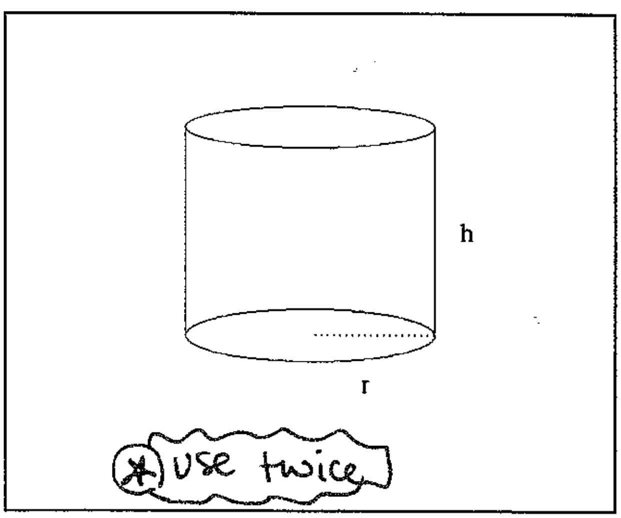
LABEL AXES w/ PROBLEM TERMINOLOGY (NOT x & y)

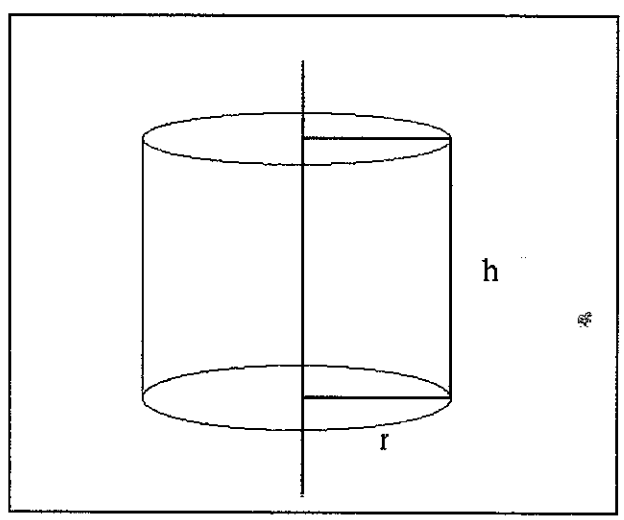

y
x
x


x/2
x/2
y
y
x


4
3
x
x
x
x
x
x
x
x


x
3 – 2x
4 – 2x


h
r
use twice


h
r

Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?

An open rectangular box with square base is to be made from 12 ft.^{2} of material. What dimensions will result in a box with the largest possible volume?

Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

A container in the shape of a right circular cylinder with no top has surface area 12π ft.^{2} What height h and base radius r will maximize the volume of the cylinder?

A cylindrical can is to hold 12π m.^{3} The material for the top and bottom costs \$12/m.^{2} and material for the side costs \$8/m.^{2} Find the radius r and height h of the most economical can.

$$V = \pi r^2 h$$

$$12\pi = \pi r^2 + (2\pi r)h$$

$$C = 12\pi r^2 + 12\pi r^2 + 8(2\pi r)h$$

$$12\pi = \pi r^2 h$$

$$V = \pi r^2 h$$

$$12 = 2r + 2h$$

$V = (4-2x)(3-2x)(x)$

none

$V = x^2 y$

$12 = x^2 + 4(xy)$

$A = x y$

$\pi x/2 + x + 2y = 12$

...: do a ...
...#1 "find words" that
indicate what you're optimizing"
highlight in 1 color. Do same for #2-6
same color: write what type of
... that would be (#2-6).
in different color: restriction...

KEY

AP Calculus AB/BC
Optimization Match Up → JUST TO SET UP

name: ______________
date: ______________

LABEL AXES w/ PROBLEM TERMINOLOGY (NOT X & Y)

1.

Construct a window in the shape of a semi-circle over a rectangle. If the distance around the outside of the window is 12 feet, what dimensions will result in the rectangle having largest possible area?

Main equation you're optimizing: rectangle area

$A = xy$

Helper equation: perimeter window

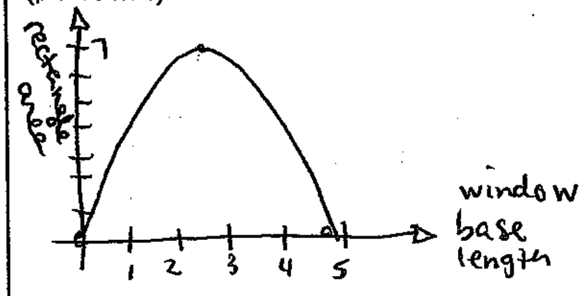
$12 = 2y + x + \pi x/2$

Optimizing equation after substitution (1 variable only):

$y = (12 - x - \pi x/2)/2$

$A = x(12 - x - \pi x/2)/2$

Good sketch of optimizing equation graph (provide scale):



2.

A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with the largest volume?

Main equation you're optimizing: box volume

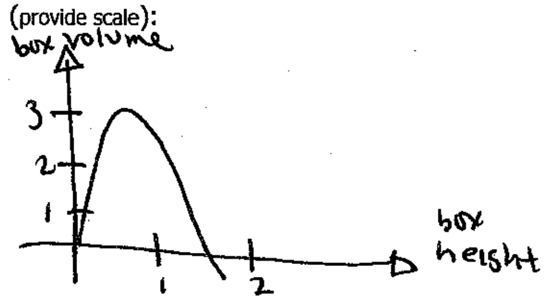
$V = (4-2x)(3-2x)(x)$

Helper equation: (none...) based on picture or orig sheet size

Optimizing equation after substitution (1 variable only):

$V = (4-2x)(3-2x)x$

Good sketch of optimizing equation graph (provide scale):



3.

An open rectangular box with square base is to be made from 12 ft^2 of material. What dimensions will result in a box with the largest possible volume?

Main equation you're optimizing: box volume

$V = x^2 y$

Helper equation: SA of box

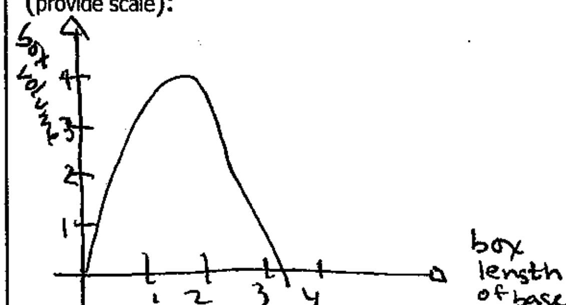
$12 = x^2 + 4xy$

Optimizing equation after substitution (1 variable only):

$\frac{12 - x^2}{4x} = y$

$V = x^2\left(\frac{12 - x^2}{4x}\right)$

Good sketch of optimizing equation graph (provide scale):



careful ... division

Good orders of operation & calculator skills arise be careful

need larger window to see behavior

LABEL AXES w/ PROBLEM TERMINOLOGY (NOT x & y)

4.

Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

Main equation you're optimizing: cylinder volume

$V = \pi x^2 y$

Helper equation: perim. of rectangle

$12 = 2x + 2y$

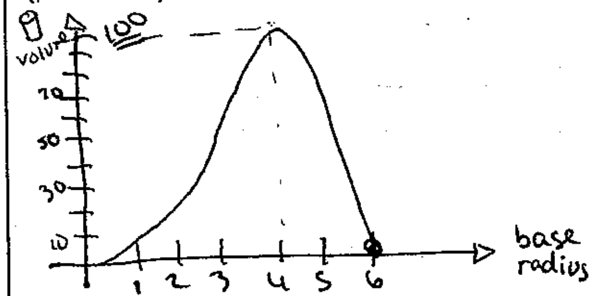
Optimizing equation after substitution (1 variable only):

$6 = x + y$

$y = 6 - x$

$V = \pi x^2 (6 - x)$

Good sketch of optimizing equation graph (provide scale):



5.

A container in the shape of a right circular cylinder with no top has surface area 12π ft^2. What height h and base radius r will maximize the volume of the cylinder?

Main equation you're optimizing: volume cylinder

$V = \pi r^2 h$

Helper equation: SA cylinder

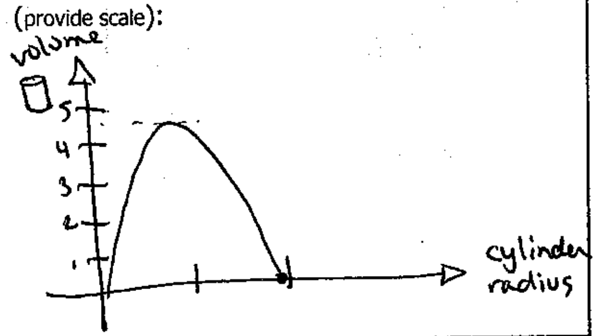
$12 = \pi r^2 + 2\pi r h$

Optimizing equation after substitution (1 variable only):

$\frac{12 - \pi r^2}{2\pi r} = h$

$V = \pi r^2 \left(\frac{12 - \pi r^2}{2\pi r} \right)$

Good sketch of optimizing equation graph (provide scale):



6.

A cylindrical can is to hold 12π m^3. The material for the top and bottom costs \$12/m^2, and material for the side costs \$8/m^2. Find the radius r and height h of the most economical can.

Main equation you're optimizing: cost

$C = 12(\pi r^2)(2) + 8(2\pi r h)$

Helper equation: volume

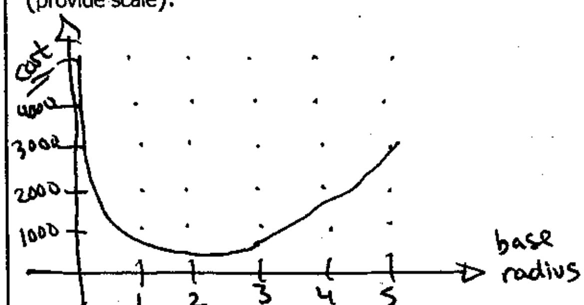
$12\pi = \pi r^2 h$

Optimizing equation after substitution (1 variable only):

$\frac{12}{r^2} = h$

$C = 24\pi r^2 + 16\pi r \left(\frac{12}{r^2} \right)$

Good sketch of optimizing equation graph (provide scale):



Need BIG window

AP Calculus AB

Integration of ln... & review... name ____________

What is the world's saddest candy?

Evaluate each expression. Cross out the letter next to each correct answer. The riddle answer will remain.

ANSWERS 1–10

S	$-\ln\sqrt{x^2+1}+C$
G	$\ln\sqrt{x^2+1}+C$
a	$3/5\ln 15$
O	$2\sqrt{x+1}+C$
T	$x^2/2-\ln x^4+C$
Z	$\frac{1}{6}\ln\lvert 6x+1\rvert+C$
O	$-3/2\ln\lvert 3-2x\rvert+C$
A	$-\frac{1}{2}\ln\lvert 3-2x\rvert+C$
W	$2\ln\lvert x-1\rvert-\frac{2}{x-1}+C$
R	$-3\ln\lvert 3-x^3\rvert+C$
T	$\ln\lvert x+1\rvert+C$
L	$1/3(\ln x)^3+C$
O	$5/3\ln 13$
D	$2/3(\ln x)^3+C$
A	$-\frac{1}{3}\ln\lvert 3-x^3\rvert+C$

ANSWERS 11–19

Z	$x+\frac{1}{2}\ln(x^2+1)+C$
Σ	$4\ln\lvert 4x-1\rvert+C$
I	$1/4\ln\lvert 4x-1\rvert+C$
P	$16/3$
U	$17/8$
M	$1/2\ln 10$
O	$15/8$
L	$x=4 \quad x=3$
R	$\frac{3\cos x}{(\sin x-1)(\sin x+2)}$
B	$\ln\lvert\ln x\rvert+C$
O	$x=0,\ x=2$
G	$14/3$
A	$(1, 1/2)$

① $\int \frac{1}{x+1}\,dx$

② $\int \frac{1}{3-2x}\,dx$

③ $\int \frac{x}{x^2+1}\,dx$

④ $\int \frac{x^2-4}{x}\,dx$

⑤ $\int \frac{(\ln x)^2}{x}\,dx$

⑥ $\int \frac{1}{\sqrt{x+1}}\,dx$

⑦ $\int \frac{2x}{(x-1)^2}\,dx$

⑧ $\int \frac{x^2}{3-x^3}\,dx$

⑨ $\int \frac{1}{6x+1}\,dx$

⑩ $\int_0^4 \frac{5}{3x+1}\,dx$

⑪ $\int \frac{1}{x\ln x}\,dx$

⑫ $\int 1+\frac{x}{x^2+1}\,dx$

⑬ $\int_0^3 \frac{x}{x^2+1}\,dx$

⑭ $\int \frac{1}{4x-1}\,dx$

⑮ Find relative extrema

$y=\frac{x^2}{2}-\ln x$

⑯ Find y' if

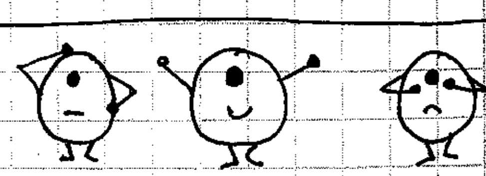
$y=\ln\left|\frac{-1+\sin x}{2+\sin x}\right|$

⑰ $\int_0^1 x(x^2+1)^3\,dx$

⑱ $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$

⑲ Find the critical numbers of

$f(x)=x^2(x-3)$



Adapted from Larson Book & Pizzazz Worksheets by Dodmehr

WHAT IS THE WORLD'S SADDEST CANDY?

① $\int \frac{1}{x+1}dx \to u=x+1,\ du=dx \to \int \frac{1}{u}du$

$\to \ln|u|+C \to \boxed{\ln|x+1|+C}$

② $\int \frac{1}{3-2x}dx \to u=3-2x,\ du=-2dx \to -\frac{1}{2}\int \frac{1}{u}du$

$-\frac{1}{2}du=dx$

$\to -\frac{1}{2}\ln|u|+C \to \boxed{-\frac{1}{2}\ln|3-2x|+C}$

③ $\int \frac{x}{x^2+1}dx \to u=x^2+1,\ du=2x\,dx \to \frac{1}{2}\int \frac{1}{u}du$

$\frac{1}{2}du=x\,dx$

$\to \frac{1}{2}\ln|u|+C \to \frac{1}{2}\ln|x^2+1|+C \to$

$\boxed{\ln\sqrt{x^2+1}+C}$

④ $\int \frac{x^2-4}{x}dx \to \int \frac{x^2}{x}-\frac{4}{x}dx \to \int x-\frac{4}{x}dx$

$\to \frac{1}{2}x^2-4\ln|x|+C \to \boxed{\frac{x^2}{2}-\ln(x^4)+C}$

⑤ $\int \frac{(\ln x)^2}{x}dx \to u=\ln x,\ du=\frac{1}{x}dx \to \int u^2du$

$\to \frac{1}{3}u^3+C \to \boxed{\frac{1}{3}(\ln x)^3+C}$

⑥ $\int \frac{1}{\sqrt{x+1}}dx \to u=x+1,\ du=dx \to \int \frac{1}{\sqrt{u}}du \to$

$\int u^{-1/2}du \to 2u^{1/2}+C \to \boxed{2\sqrt{(x+1)}+C}$

⑦ TRICKY $\int \frac{2x}{(x-1)^2}dx \to u=x-1,\ du=dx \to u+1=x,\ 2(u+1)=2x$

$\int \frac{2(u+1)}{u^2}du \to \int \frac{2u}{u^2}+\frac{2}{u^2}du \to$

$\int 2u^{-1}+2u^{-2}du \to 2\ln|u|+-2u^{-1}+C$

$\to \boxed{2\ln|x-1|-\frac{2}{x-1}+C}$

⑧ $\int \frac{x^2}{3-x^3}dx \to u=3-x^3,\ du=-3x^2dx \to -\frac{1}{3}\int \frac{1}{u}du$

$-\frac{1}{3}du=x^2dx$

$\to -\frac{1}{3}\ln|u|+C \to \boxed{-\frac{1}{3}\ln|3-x^3|+C}$

⑨ $\int \frac{1}{6x+1}dx \to u=6x+1,\ du=6dx \to \frac{1}{6}\int \frac{1}{u}du$

$\frac{1}{6}du=dx$

$\to \frac{1}{6}\ln|u|+C \to \boxed{\frac{1}{6}\ln|6x+1|+C}$

⑩ $\int_0^4 \frac{5}{3x+1}dx \to u=3x+1,\ du=3dx \to \frac{1}{3}\int \frac{5}{u}du \to$

$\frac{1}{3}du=dx$

$\frac{5}{3}\ln|u| \to \frac{5}{3}\ln|3x+1|\Big|_0^4 \to$

$\frac{5}{3}\ln|12+1|-\frac{5}{3}\ln|0+1| = \boxed{\frac{5}{3}\ln(13)}-0$

⑪ $\int \frac{1}{x\ln x}dx \to u=\ln x,\ du=\frac{1}{x}dx \to \int \frac{1}{u}du \to$

$\ln|u|+C \to \boxed{\ln|\ln x|+C}$

⑫ $\int 1+\frac{x}{x^2+1}dx = \int 1\,dx+\underbrace{\int \frac{x}{x^2+1}dx}_{\text{like \#3}}$

$\to \boxed{x+\ln\sqrt{x^2+1}+C}$

⑬ $\int_0^3 \frac{x}{x^2+1}dx = \frac{1}{2}\ln|x^2+1|\Big|_0^3$ (like #3)

$\to \frac{1}{2}[\ln 10-\ln 1] \to \boxed{\frac{1}{2}\ln 10}$ or $\ln\sqrt{10}$

⑭ $\int \frac{1}{4x-1}dx \to u=4x-1,\ du=4dx \to \frac{1}{4}\int \frac{1}{u}du$

$\frac{1}{4}du=dx$

$\to \frac{1}{4}\ln|u|+C \to \boxed{\frac{1}{4}\ln|4x-1|+C}$

(15) $y' = \frac{2x}{2} - \frac{1}{x} \rightarrow y' = x - \frac{1}{x}$

$0 = x - \frac{1}{x} \rightarrow \boxed{x=0}$ critical # (why?)

& $\frac{1}{x} = x \rightarrow 1 = x^2 \rightarrow \boxed{x = \pm 1}$

$y = \frac{x^2}{2} - \ln x \rightarrow$ so not defined at $\underline{x=0}$ or $\underline{x=-1}$ (why?)

so check $\underline{x=1}$

$y'(\tfrac{1}{2})<0 \quad 1 \quad y'(2)>0$

$\therefore$ minimum at $x=1$

point is $(1, \frac{1}{2})$ plug $x=1$ into orig equation

(16) $y = \ln(-1+\sin x) - \ln(2+\sin x)$

$$y' = \frac{\cos x}{-1+\sin x} - \frac{\cos x}{2+\sin x}$$

$$= \frac{(2+\sin x)(\cos x) - (-1+\sin x)(\cos x)}{(-1+\sin x)(2+\sin x)}$$

$$= \frac{2\cos x + \cancel{\sin x \cos x} + \cos x - \cancel{\sin x \cos x}}{(-1+\sin x)(2+\sin x)}$$

$$= \boxed{\frac{3\cos x}{(\sin x - 1)(2+\sin x)}}$$

(17) $\int_0^1 x(x^2+1)^3\,dx \rightarrow$ $u = x^2+1$, $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^3\,du \rightarrow \frac{1}{4}\cdot\frac{1}{2}u^4 \rightarrow$

$\frac{1}{8}(x^2+1)^4\Big|_0^1 \rightarrow \frac{1}{8}\left[(1+1)^4 - (0+1)^4\right]$

$= \frac{1}{8}\left[16 - 1\right] = \boxed{15/8}$

TRICKY!

(18) $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx \rightarrow$ $u = 2x-1$, $du = 2\,dx$, $\frac{1}{2}du = dx$, $\rightarrow \frac{u+1}{2} = x$

$\frac{1}{2}\int_1^9 \frac{\frac{u+1}{2}}{\sqrt{u}}\,du \rightarrow \frac{1}{2}\int_1^9 \frac{u+1}{2\sqrt{u}}\,du \rightarrow$

$\frac{1}{4}\int_1^9 \frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}}\,du \rightarrow \frac{1}{4}\int_1^9 u^{1/2} + u^{-1/2}\,du$

$\rightarrow \frac{1}{4}\left[\frac{2}{3}u^{3/2} + \frac{2}{1}u^{1/2}\right]\Big|_1^9$

$\rightarrow \frac{1}{4}\left[\left(\frac{2}{3}(9)^{3/2} + 2(\sqrt{9})\right) - \left(\frac{2}{3}(1) + 2(1)\right)\right]$

$\rightarrow \frac{1}{4}\left[\frac{2}{3}(27) + 6 - \frac{2}{3} - 2\right]$

$\rightarrow \frac{1}{4}\left[18 + 6 - \frac{2}{3} - 2\right] \rightarrow \frac{1}{4}\left(\frac{64}{3}\right)$

$\rightarrow \boxed{16/3}$

(19) $f'(x) = 2x(x-3) + x^2$

$= 2x^2 - 6x + x^2$

$= 3x^2 - 6x$

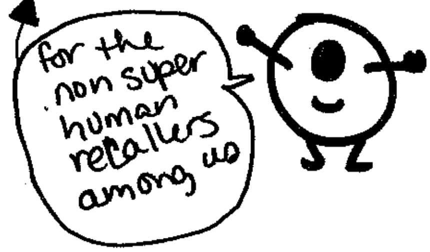
$= 3x(x-2)$

$\boxed{x=0 \quad x=2}$

check

$f'(-1) \quad 0 \quad f'(1)<0 \quad 2 \quad f'(3)>0$

e^x $\ln x$ basics refresher



① $e^{2x+\ln 3} =$ ________________

② $e^{5+\ln x} =$ ________________

③ $e^{\ln 7+\ln x^2} =$ ________________

④ $e^{x \cdot \ln 5} =$ ________________

⑤ $e^{2 \cdot \ln 12} =$ ________________

⑥ $e^{7 \cdot \ln x} =$ ________________

⑦ $e^{\ln 5-\ln x} =$ ________________

⑧ $e^{12-\ln 2} =$ ________________

⑨ $\ln(e^{12} \cdot 7) =$ ________________

⑩ $\ln\left(e^7/x^3\right) =$ ________________

⑪ $\ln\left(e^8+e^x\right) =$ ________________

(12) solve for B $\ln B = 12 + \ln 8$

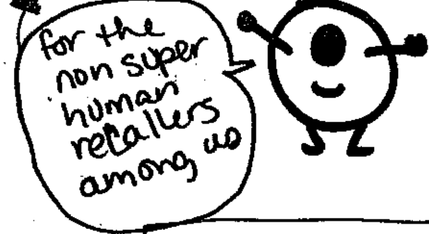
(13) solve for B $\ln B = x - \ln 10$

(14) solve for B $e^B = x^2 \cdot e^5$

(15) solve for B $e^{2B} = x^3 + 7$

(16) solve for B $e^{7B+2} = 5x - 2e^3$

e^x $\ln x$ basics refresher



(1) $e^{2x+\ln 3} = e^{2x}\cdot e^{\ln 3} = 3e^{2x}$

(2) $e^{5+\ln x} = e^5 e^{\ln x} = e^5\cdot x = xe^5$

(3) $e^{\ln 7+\ln x^2} = e^{\ln 7}e^{\ln x^2} = 7\cdot x^2 = 7x^2$

(4) $e^{x\cdot\ln 5} = (e^x)^{\ln 5} = (e^{\ln 5})^x = 5^x$

(5) $e^{2\cdot\ln 12} = (e^{\ln 12})^2 = 12^2 = \boxed{144}$

(6) $e^{7\cdot\ln x} = (e^{\ln x})^7 = \boxed{x^7}$

(7) $e^{\ln 5-\ln x} = \dfrac{e^{\ln 5}}{e^{\ln x}} = \boxed{5/x}$

(8) $e^{12-\ln 2} = \dfrac{e^{12}}{e^{\ln 2}} = e^{12}/2 = \frac{1}{2}e^{12}$

(9) $\ln(e^{12}\cdot 7) = \ln(e^{12}) + \ln 7 = \boxed{12+\ln 7}$

(10) $\ln\left(e^7/x^3\right) = \ln e^7 - \ln x^3 = \boxed{7-3\ln x}$

(11) $\ln(e^8+e^x) = \ln(e^8+e^x)$

$\cancel{e^{8+x}}$

be **careful**

$$x^a\cdot x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$x^{-a} = \frac{1}{x^a}$$

$$\frac{1}{x^{-a}} = x^a$$

$$x^{1/5} = \sqrt[5]{x}$$

$$x^{-1/3} = \frac{1}{\sqrt[3]{x}}$$

(12) solve for B $\qquad \ln B = 12 + \ln 8$

$$e^{\ln B} = e^{12+\ln 8} \longrightarrow B = e^{12} e^{\ln 8}$$

e to the whole side ....

$$B = e^{12} \cdot 8 \qquad \boxed{B = 8e^{12}}$$

(13) solve for B $\qquad \ln B = x - \ln 10$

$$e^{\ln B} = e^{x - \ln 10}$$

$$B = e^x / e^{\ln 10} \longrightarrow \boxed{B = \frac{e^x}{10}} \text{ or } \boxed{B = \tfrac{1}{10} e^x}$$

---

(14) solve for B $\qquad e^B = x^2 \cdot e^5$

$$\ln e^B = \ln(x^2 \cdot e^5)$$

$$B = \ln x^2 + \ln e^5 \longrightarrow \boxed{B = 2\ln(x) + 5}$$

or $\boxed{B = \ln(x^2) + 5}$

---

(15) solve for B $\qquad e^{2B} = x^3 + 7$

$$\ln(e^{2B}) = \ln(x^3+7)$$

$$2B = \ln(x^3+7)$$

$\boxed{B = \frac{\ln(x^3+7)}{2}}$ or $\boxed{B = \tfrac{1}{2}\ln(x^3+7)}$

or $\boxed{B = \ln\sqrt{(x^3+7)}}$

---

(16) solve for B $\qquad e^{7B+2} = 5x - 2e^3$

$$\ln(e^{7B+2}) = \ln(5x - 2e^3)$$

true but ....

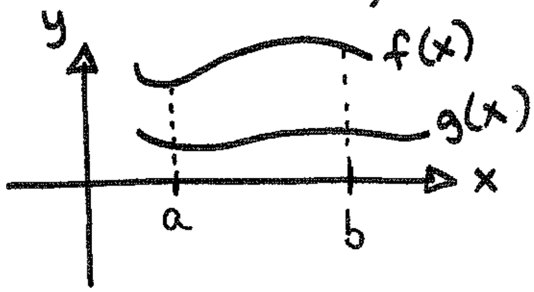
$$\ln\left(\cancel{e^{7B}e^{2}}\right) = \quad "$$

$$\cancel{7B+2} = \frac{\ln(5x - 2e^3) - 2}{7}$$

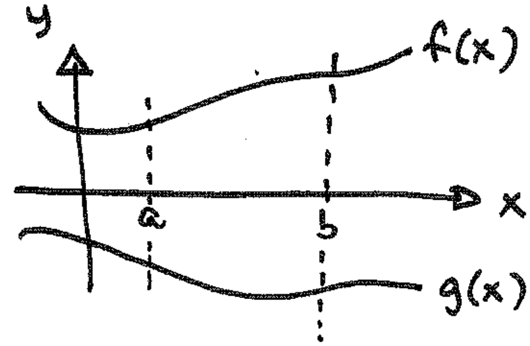
$$B =$$

Curvy Area

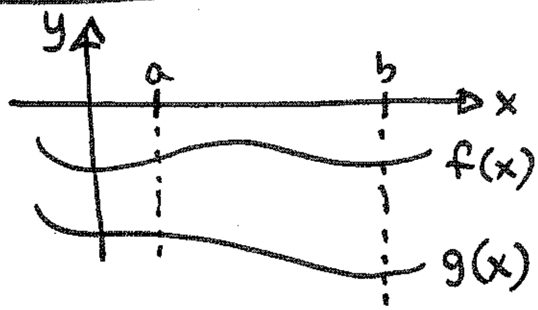
(area between curves)



①



②

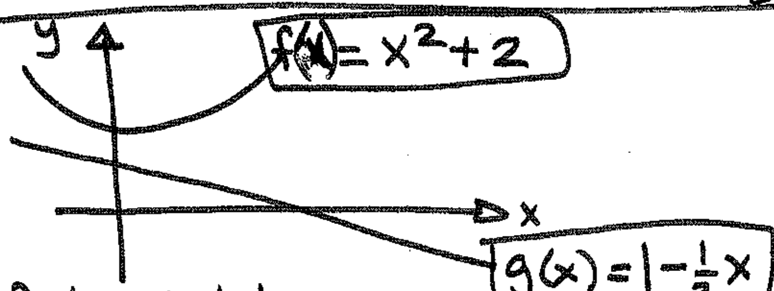


③

Moral of the Story

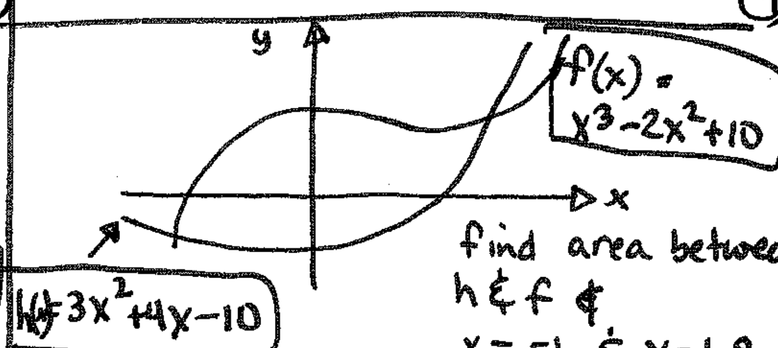
Area between 2 curves: from a to b...

④



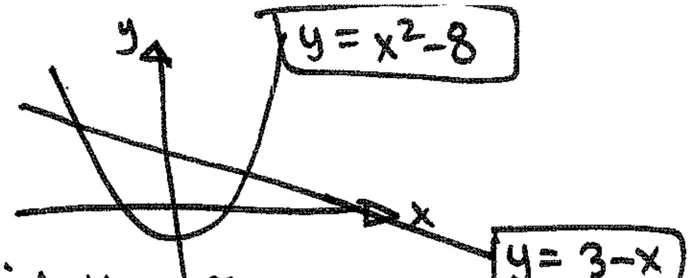
find area between $f(x)$ & $g(x)$ & y-axis and $x = 1.5$

⑤



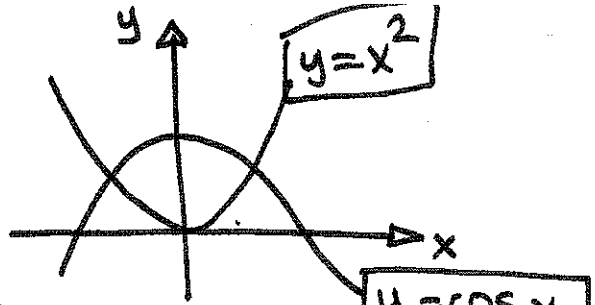
find area between h & f & $x = -1$ & $x = 1.8$

⑥



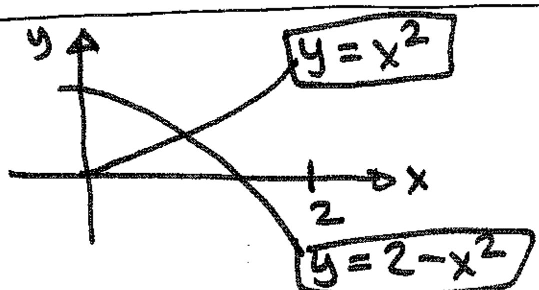
Find the area between the 2 curves

7



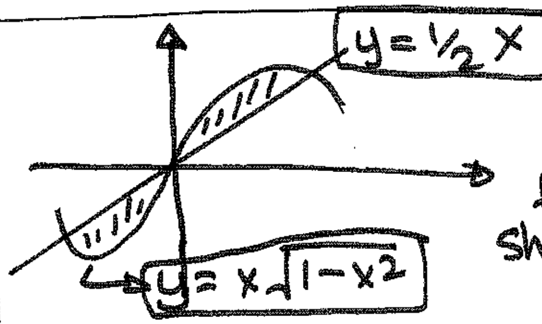
find the area between the 2 curves

8



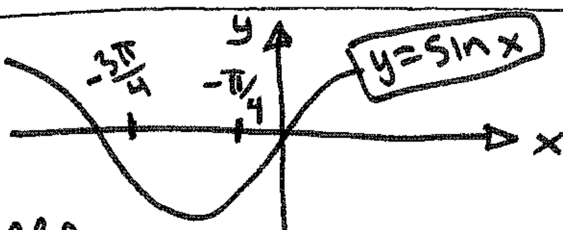
Find the area bounded by curves & $0 \le x \le 2$

9



find shaded area

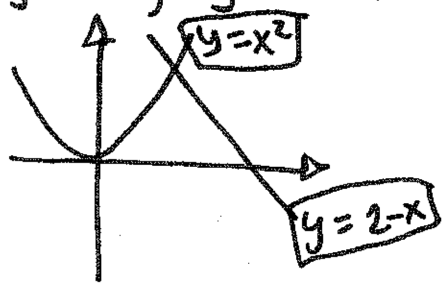
10



find area from $-\frac{3\pi}{4} \le x \le \frac{-\pi}{4}$, x axis, & $y=\sin x$

11

Find area bounded by $y=x^2$, $y=2-x$ & $y=0$.



12

Compute the area between curves $y = 2 - x^2$ & $y = -2$

13

Compute the area between $y = 3x^2 + 12$ & $y = 4x + 4$ & $x = -3$ & $x = 3$

14

Find the area bounded by $y = x^2$ & $y = x^3$

15

Find the area bounded by $y = x^2$ & $y = 6 - x$

16

Find the area bounded by $f(x) = x^2 - 4x + 3$ & $g(x) = -x^2 + 2x + 3$

17

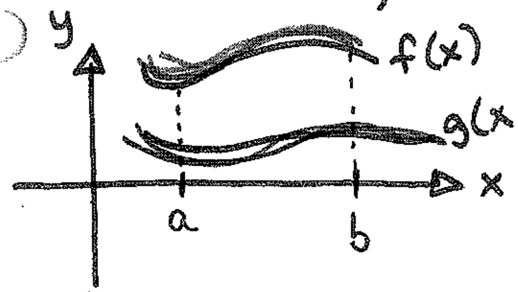
Find the area bounded by $y = x^2 + 2x + 1$ & $y = 3x + 3$

18

IMPORTANT TO REMEMBER

Curvy Area

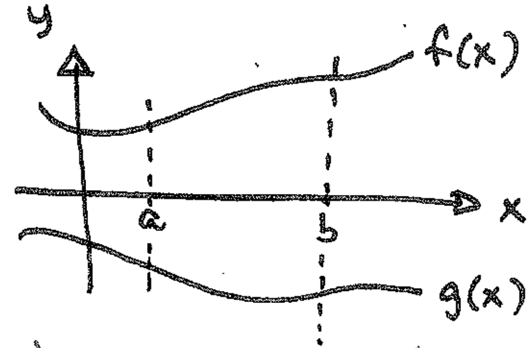
(area between curves)



$$\int_a^b f(x) - g(x)\ dx$$

→ exact area between f & g from a to b

(1)

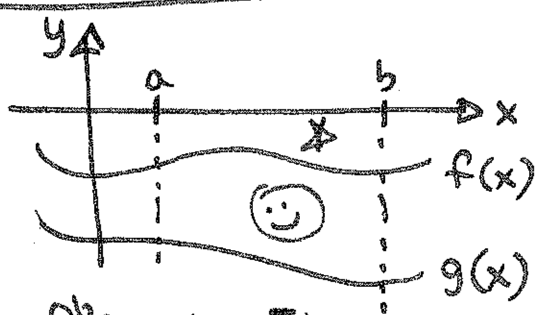


$$\int_a^b f(x)\,dx - \underbrace{\int_a^b g(x)\,dx}_{\text{since negative}}$$

why?

$$\boxed{\int_a^b f(x) - g(x)\,dx}$$

(2)



$\int_a^b f(x)\,dx = ☆$

$\int_a^b g(x)\,dx = -☆ + -☺$

$$= \boxed{\int_a^b f(x) - g(x)\ dx}$$

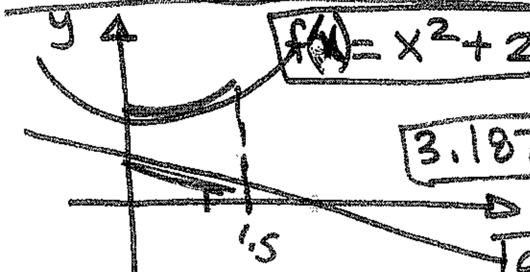
(3)

moral of the Story

Area between 2 curves: from a to b ...

$$\int_a^b \text{top curve} - \text{bottom curve}\ dx$$

(4)



$\boxed{f(x) = x^2 + 2}$ $\boxed{3.1875}$ on calc.

$\boxed{3.187}$, $\boxed{3.188}$

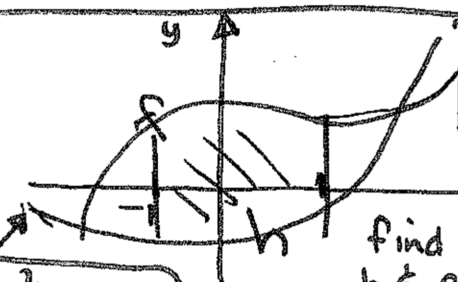
$\boxed{g(x) = 1 - \frac{1}{2}x}$

find area between f(x) & g(x) & y-axis and x = 1.5

$$\int_0^{1.5} (x^2+2) - (1 - \tfrac{1}{2}x)\ dx$$

$$\int_0^{1.5} x^2 + 1 + \tfrac{1}{2}x\ dx$$

(5)



$\boxed{f(x) = x^3 - 2x^2 + 10}$

$\boxed{h(x) = 3x^2 + 4x - 10}$

find area between h & f & x = -1 & x = 1.8

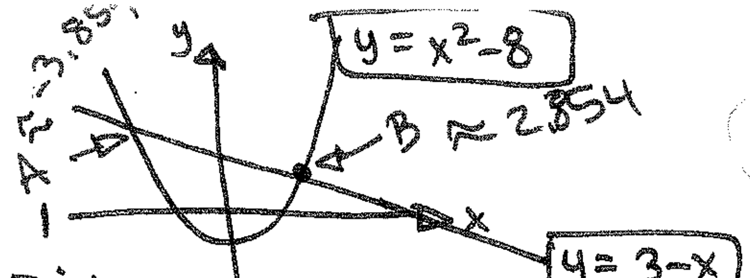
$$\int_{-1}^{1.8} f - h\ dx \approx 42.507$$

or 42.508 or ....

$Y_3 = Y_1 - Y_2$

f h

(6)

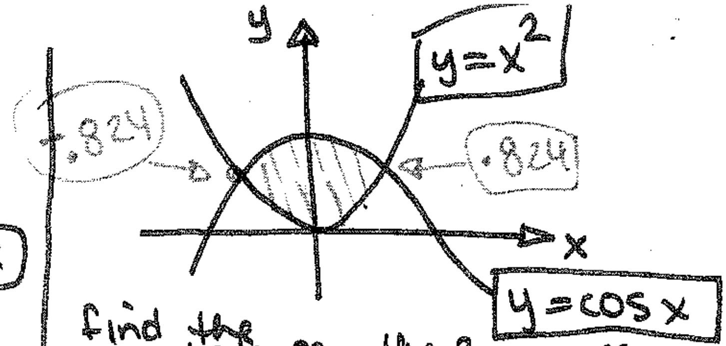


Find the area between the 2 curves

$$\int_A^B (3-x) - (x^2-8)\,dx$$

≈ 50.311
or 50.312

(7)

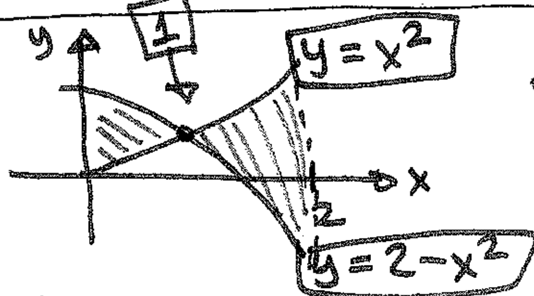


find the area between the 2 curves

must be in radians

$\boxed{1.095}$

hwk (8)



$\boxed{4}$

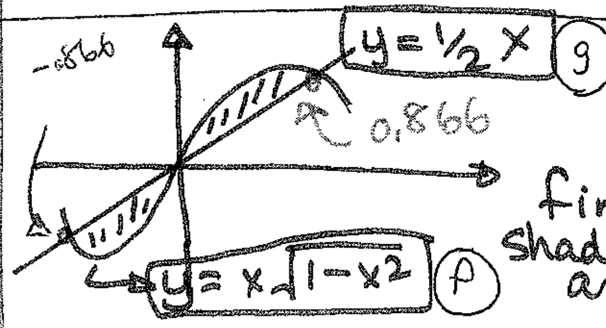
Find the area bounded by curves & $0 \le x \le 2$

$$\int_0^1 (2-x^2) - x^2\,dx \; +$$

$$\int_1^2 x^2 - (2-x^2)\,dx$$

$$\int_0^1 2-2x^2\,dx + \int_1^2 2x^2-2\,dx$$

(9)

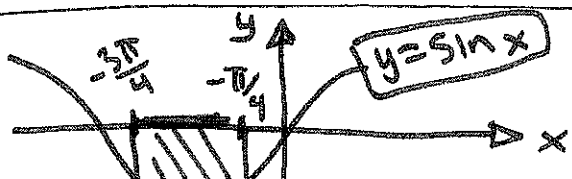


find shaded area

$\boxed{0.208}$

$$\int_{-0.866}^{0} g-f\,dx + \int_0^{0.866} f-g\,dx$$

hwk (10)



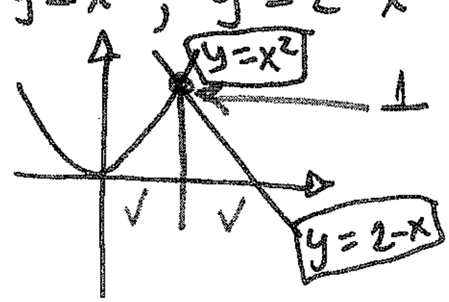
find area from $-\frac{3\pi}{4} \le x \le -\frac{\pi}{4}$, x axis, & $y = \sin x$

$$\int_{-\frac{3\pi}{4}}^{-\pi/4} 0 - \sin x\,dx$$

$\approx \boxed{1.414}$

(11)

Find area bounded by $y=x^2$, $y=2-x$ & $y=0$.

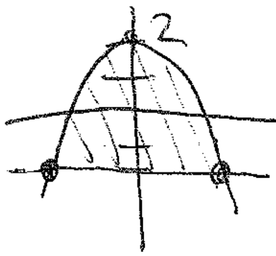


$$\int_0^1 x^2\,dx + \int_1^2 2-x\,dx$$

$0.8\overline{3}$ .....

(12)

Compute the area between curves $y = 2 - x^2$ & $y = -2$



$-2 = 2 - x^2$

$x^2 = 4$

$x = \pm 2$

$\boxed{10.667}$

$10.\overline{6}$

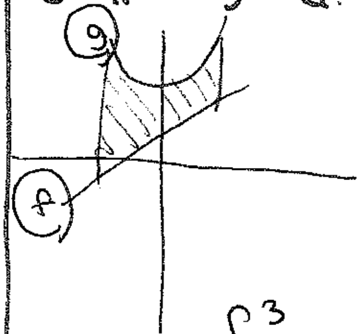
10.666

10.666

hw

(13)

Compute the area between $y = 3x^2 + 12$ & $y = 4x + 4$ & $x = -3$ & $x = 3$



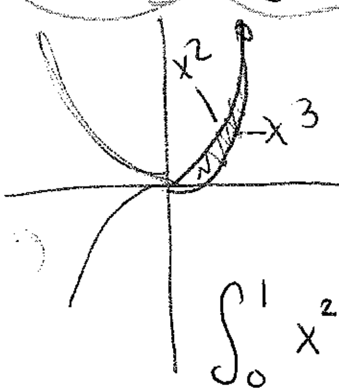
$\boxed{102}$

$$\int_{-3}^{3} g - f \, dx$$

hw

(14)

Find the area bounded by $y = x^2$ & $y = x^3$



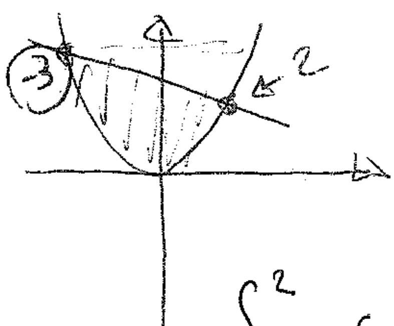
$\boxed{0.083}$

$$\int_0^1 x^2 - x^3 \, dx$$

hw

(15)

Find the area bounded by $y = x^2$ & $y = 6 - x$

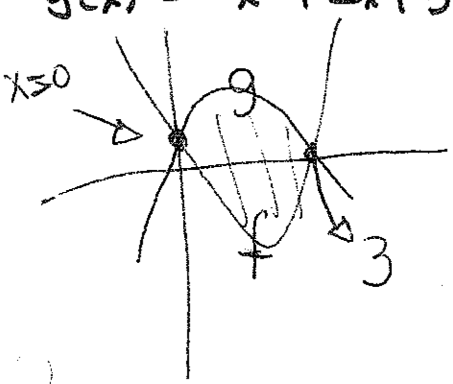


$\boxed{20.833}$

$$\int_{-3}^{2} (6 - x) - x^2 \, dx$$

(16)

Find the area bounded by $f(x) = x^2 - 4x + 3$ & $g(x) = -x^2 + 2x + 3$



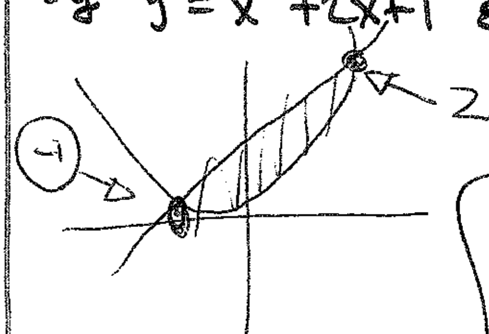
$\boxed{9}$

$$\int_0^3 -2x^2 + 6x \, dx$$

$-\tfrac{2}{3}x^3 + 3x^2 \Big|_0^3 = \quad -18 + 27$

(17)

Find the area bounded by $y = x^2 + 2x + 1$ & $y = 3x + 3$



$\boxed{4.5}$

$$\int_{-1}^{2} (3x + 3) - (x^2 + 2x + 1) \, dx$$

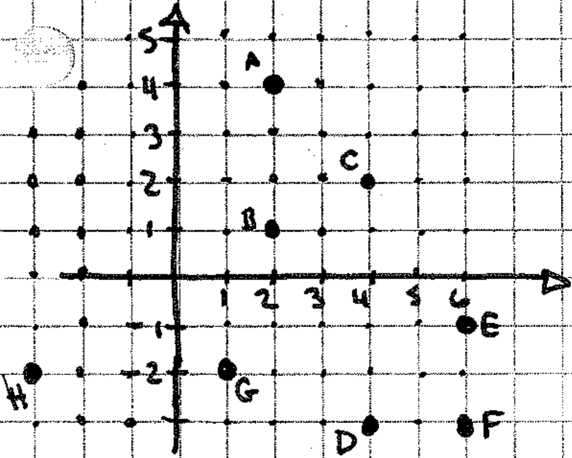
(18)

IMPORTANT TO REMEMBER

fnInt(f(x) , X , a, b) → $\int_1^7 x^2+1\,dx$

$\int_a^b f(x)\,dx$ "x" fnInt(x^2+1, x, 1, 7)

Distance between 2 Points (Length)



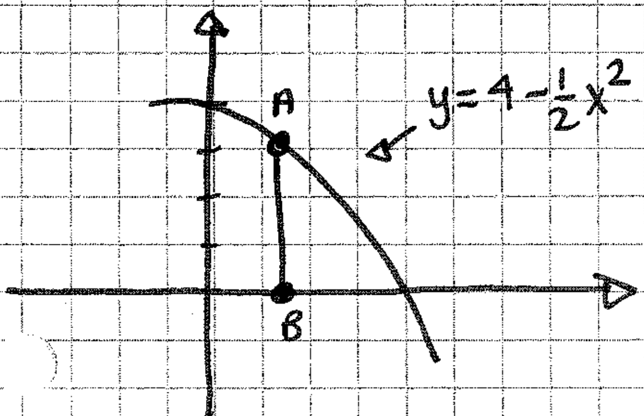
AB =

CD =

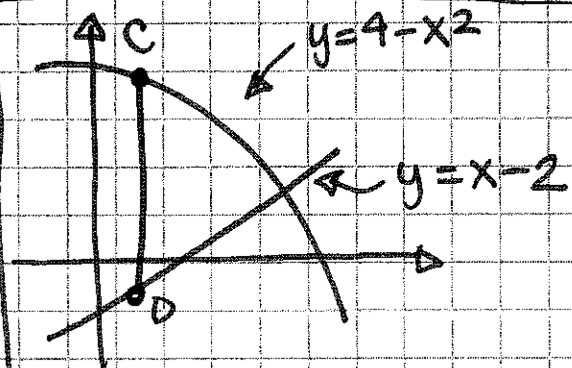
EF =

GH =

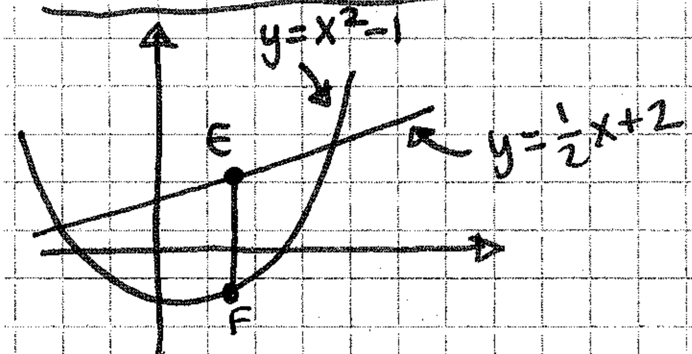
DF =



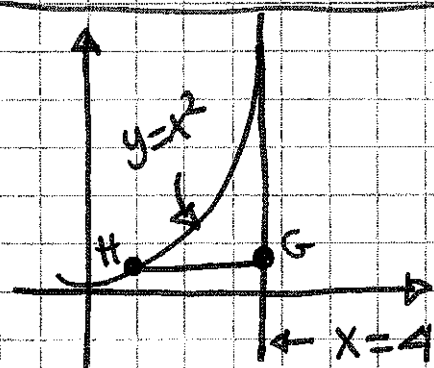
AB =



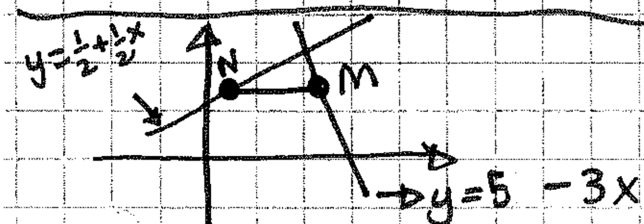
CD =



EF =



GH =



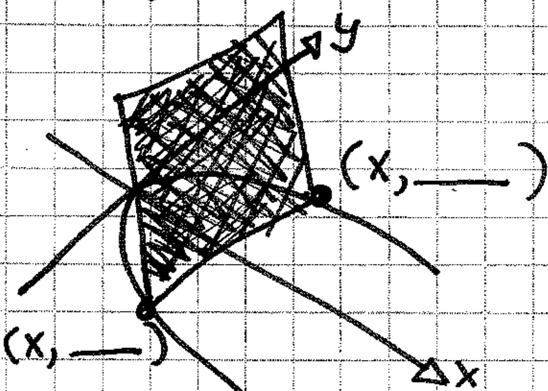
MN =

AB =

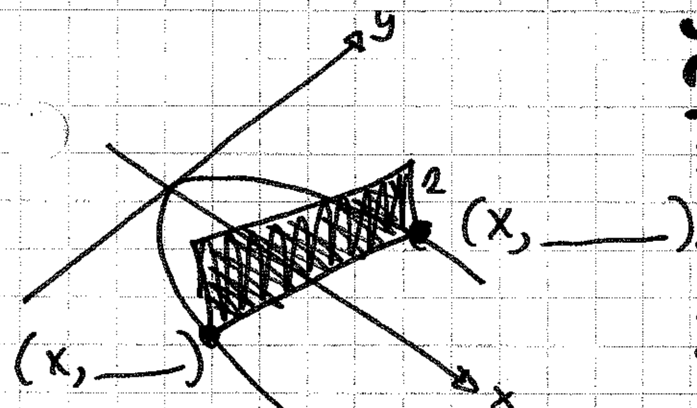
CROSS SECTION VOLUME

for 1–3 Let R be the region bounded by $\boxed{x=y^2}$ & $\boxed{x=9}$. Find the volume of the solid that has R as its base & if every cross section by a plane ⊥ to x-axis has given shape.

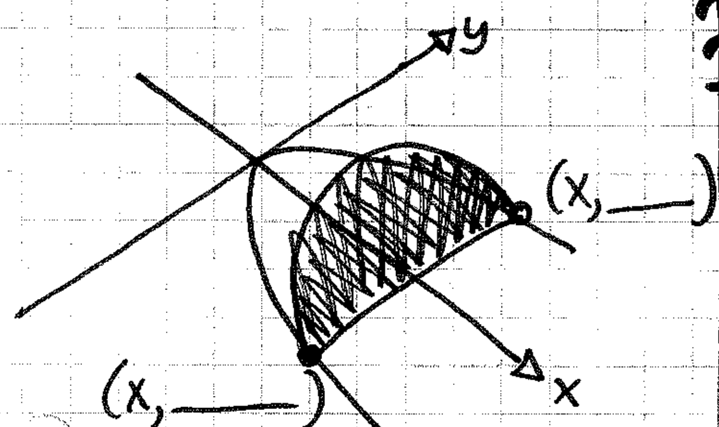
① A square



② A rectangle of height 2

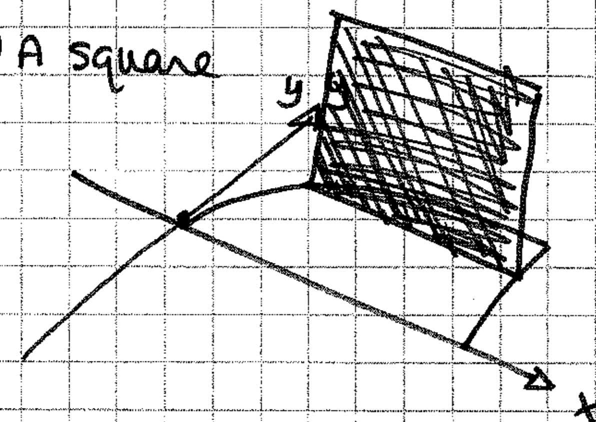


③ A semicircle

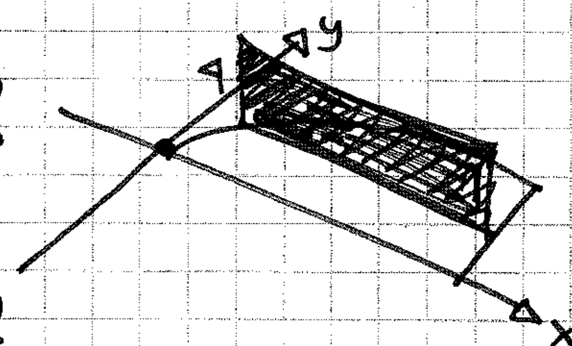


for 4–6 Let R be the region bounded by $\boxed{y=\sqrt{x}}$ $\boxed{x=9}$ & $\boxed{y=0}$. Find the volume of the solid that has R as its base if every cross section by a plane ⊥ to the y-axis has given shape.

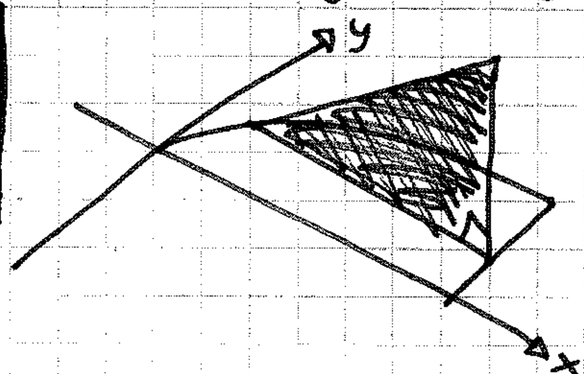
④ A square



⑤ A rectangle of height 4.



⑥ A right triangle w/ one leg on R & one leg of length **6.**



USE A SEPARATE SHEET

KEY

Cross Section Volume

1. $x=y^2 \quad x=9$ find volume $\int_0^9 4x\,dx$

$V=\int_0^9 (2\sqrt{x})(2\sqrt{x})\,dx \qquad y=\sqrt{x} \qquad \sqrt{x}+\sqrt{x} \qquad 2x^2\Big|_0^9 \qquad 2(9)^2=2(81)=\boxed{162}$

2. $x=y^2 \quad x=9 \qquad R=\sqrt{x}+\sqrt{x} \quad R=2\sqrt{x}$

$y=\sqrt{x}$

$V=\int_0^9 (2\sqrt{x})(2)\,dx \quad \int_0^9 4\sqrt{x} \quad 4x^{1/2} \quad \frac{8}{3}x^{3/2}\Big|_0^9 \quad \frac{8}{3}(9)^{3/2}=\boxed{72}$

3. $V=\int_0^9 \frac{\pi(\sqrt{x})^2}{2}\,dx \qquad y=\sqrt{x} \qquad 2\sqrt{x} \quad r=\sqrt{x}$

$\frac{\pi}{2}\int_0^9 x\,dx$

$\frac{\pi}{2} \quad \frac{1}{2}x^2 \quad \frac{\pi}{2}\cdot\frac{1}{2}(9)^2 \qquad \frac{\pi}{2}\cdot\frac{81}{2}=\boxed{\frac{81\pi}{4}}$

4. $y=\sqrt{x} \quad x=9 \quad y=0$

$x=y^2 \qquad (9-y^2)(9-y^2)$

$V=\int_0^{\sqrt{9}} (9-y^2)^2\,dy \qquad 81-9y^2-9y^2+y^4$

$V=\int_0^{\sqrt{9}} (y^4-18y^2+81)\,dy$

$\frac{1}{3}y^3-\frac{18}{2}y^2+81y \qquad \frac{1}{3}y^3-\frac{18}{2}y^2+81y\Big|_0^3 \qquad \frac{1}{3}(3)^3-\frac{18}{2}(3)^2+81(3)$

$9-81+243 \quad 72 \quad \boxed{171}$

5. $y=\sqrt{x} \quad x=9 \quad y=0 \qquad x=y^2$

$V=\int_0^3 (9-y^2)(4)\,dy$

$V=\int_0^3 36-4y^2$

$36y-\frac{4}{3}y^3 \qquad 36(3)-\frac{4}{3}(3)^3=108-36=\boxed{72}$

108

6. $y=\sqrt{x} \quad x=9 \quad y=0$

$V=\int_0^3 \frac{(9-y^2)(6)}{2}\,dy$

$V=\int_0^3 \frac{(54-6y^2)}{2}\,dy$

$V=\int_0^3 27-3y^2\,dy$

$27y-y^3$

$27y-y^3\Big|_0^3$

$27(3)-(3)^3$

$81-27$

$\boxed{54}$