Prior to Calculus based on 2013 AB 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by the continuous function $G(t)$, where t is measured in hours and $0 \leq t \leq 10$. Selected values of $G(t)$ are given in the table below. At the beginning of the workday (t = 0), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 10$, the plant processes gravel at a constant rate of 100 tons per hour.

t (hours)	0	4	6	7	10
$G(t)$ (tons per hour)	135	120	70	50	125

a) Find the average rate of change of $G(t)$ over the time interval $4 \leq t \leq 6$. Show the calculations that lead to your answer. Using correct units, interpret your answer in the context of the problem.

b) Is the amount of unprocessed gravel at the plant increasing or decreasing at time t = 6 hours? Explain your reasoning.

c) Based on the given mathematical model, is there a time where gravel arrives at the gravel processing plant at a rate of 100 tons per hour? Explain your reasoning.

d) Assuming that gravel arrives at the plant at a constant rate of 135 tons per hour for the first four hours of operation, how much unprocessed gravel arrives at the plant during that time interval?

e) Using data from the table and the sum of four products, approximate the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday. Show the calculations that lead to your answer.

f) Using your answer from part e, approximate the amount of unprocessed gravel at the plant at the end of the workday (t = 10).

Changing the Context to Create a Parallel Problem

On a certain workday, the rate, in applications per hour, at which unprocessed applications arrive at a government processing office is modeled by $G(t) = 60 + 30\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday (t = 0), the office has 150 unprocessed applications. During the hours of operation, $0 \le t \le 8$, the office processes applications at a constant rate of 50 applications per hour.

a) Find $G'(6)$. Using correct units, interpret your answer in the context of the problem.

b) Find the total number of unprocessed applications that arrive in the office during the hours of operation on this workday. Round your answer to the nearest whole number.

c) Is the number of unprocessed applications in the office increasing or decreasing at $t = 6$ hours? Show the work that leads to your answer.

d) What is the maximum amount of unprocessed applications in the office during the hours of operation on this workday? Justify your answer.

Expanding the Original Problem

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday (t = 0), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

b) **Find the critical point of *G*(*t*) and interpret the significance of this point in the context of this problem.**

c) **Write an equation for the line tangent to $G(t)$ at the point (5, 98.141). Use the tangent line to approximate *G*(5.5).**

d) **Given that *G″*(5) = -7.430, does your tangent line approximation in part b overestimate or underestimate *G*(5.5)?**

e) **Based on information found or given so far, determine which of the following statements is true about *G*(t) at t = 5. Explain your reasoning.**

   i. ***G* is increasing at a decreasing rate.**
   ii. ***G* is increasing at an increasing rate.**
   iii. ***G* is decreasing at a decreasing rate.**
   iv. ***G* is decreasing at an increasing rate.**

f) **Find the average rate of change of G(t) over the time interval $0 \le t \le \sqrt{9\pi}$. Show the calculation that leads to your answer.**

g) **Is there a time in the interval $0 \le t \le \sqrt{9\pi}$ where *G* '(t) equals your answer from part f? Justify your answer.**

h) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

i) **At what constant rate would gravel need to be processed so there is no unprocessed gravel left at the plant at the end of the workday?**

j) **Using correct units, interpret the meaning of $\frac{1}{8}\int_0^8 G(t)\,dt$ in the context of the problem.**

k) Is the amount of unprocessed gravel at the plant increasing or decreasing at *t* = 5 hours? Show the work that leads to your answer.

l) **Write an integral expression for *A*(*t*), the amount of unprocessed gravel at the plant at any time *t*.**

m) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

2013 AB 6

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

a) **The point (2, - ln3) is a critical point of *f*(*x*). Is this point a local maximum, a local minimum or neither? Justify your answer.**

b) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

c) **Is your tangent line approximation from part b an underestimate or overestimate of the actual value of *f* (1.2)? Explain your reasoning.**

d) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

e) **Avery, the ace calculus student, claims that there is a horizontal tangent line to *f* (*x*) at x = 0 since $dy/dx = 0$ at that point. Explain why this is a misconception.**

2012 AB 6

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

a) **Find all values of t in the interval $0 \le t \le 12$ for which the speed of the particle is ½.**

b) For $0 \le t \le 12$, when is the particle moving to the left?

c) **Write an expression for $x(t)$, the position of the particle at any time t, that does not include an integral.**

d) **For what value of t is the particle farthest to the right? Justify your answer.**

e) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

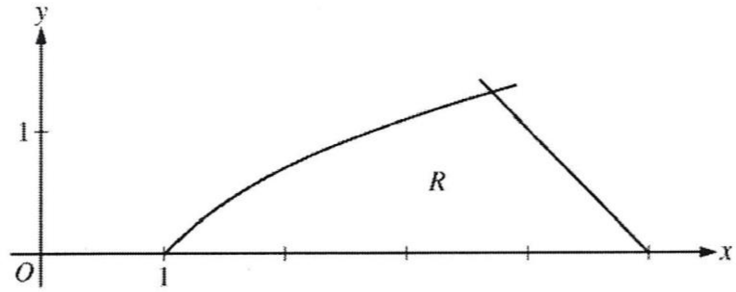
f) **Write, but do not evaluate an integral expression that gives the average speed of the particle over the interval $0 \le t \le 12$.**

g) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

h) Find the position of the particle at time $t = 4$.

i) **A second particle has position given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 5t + 7$. During what time intervals are the two particles moving in opposite directions?**

2012 AB 2



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

b) Find the volume of the solid formed by revolving region R around the x-axis.

c) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

d) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is an equilateral triangle. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

e) Find the volume of the solid formed by revolving region R around the horizontal line y = 2.

f) Find the volume of the solid formed by revolving region R around the vertical line x = 5.

g) Region R is the base of a solid. For the solid, each cross section perpendicular to the y-axis is a rectangle with base in region R and a height of 2. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

h) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

i) (BC only) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R.