Cedric French-Galan nfrench@houstonisd.org

Preparing students for the AP Calculus Exam during the school year

Websites:

- Mastermathmentor.com
- Edmodo.com
- Khanacademy.org
- http://www.ncaapmt.org/calculus/
- http://apcentral.collegeboard.com/apc/Controller.jpf
- http://designatedderiver.wikispaces.com/
-

BC 2011-2012

August 22 – September 30
First Six Weeks – all dates are subject to change

Monday	Tuesday	Wednesday	Thursday	Friday
August 22 Info sheet Name plate Syllabus POW- journal	23 Donated Items syllabus quiz TI 89 Info Discuss summer packet autobiography	24 Review Limits and Derivatives	25	26 Summer packet test turn in binder
29 Implicit diff, (odds) POW- study group picture Journal	30 Related rates skit, notes (1-25 odds, 26)	31 Finish	September 1	2 Straight line motion (odds)
5 *Labor Day Holiday* 	6 Finish POW journal	7 Rolle's thm. And mean value thm.	8	9 Celebration up to related rates
12 Function analysis POW journal	13 Finish	14 Finding absolute extreme	15	16 Approximate using diff.
20 Optimization POW journal	21 Optimization finish (all)	22 Indefinite integration	23	24 Fiesta up To Mean Value Thm.
26 slope fields POW Journal	27 U-Sub	28 Sigma Notation	29 Fall Holiday 	30 Area Under Curve

NOTE:

AP® Instructional Planning Report – By Section (2011)

Print / Download Options

✔ Data Updated Jul 8, 2011, Report Run Jul 9, 2011

Westside High School (443522) - Calculus BC - Section Not Designated

Performance on Multiple-Choice Section (Maximum Possible Score = 54)

					Number of Students in Your Group			
Content Area	Number of Questions	Global Mean	Group Mean		Lowest Fourth	Second Fourth	Third Fourth	Highest Fourth
CALCULUS AB TOPICS	27	17.2	18.2	1.058	6	9	17	8
CALCULUS BC-ONLY TOPICS	18	11.2	10.5	.9375	11	15	5	9
DIFFERENTIAL CALCULUS	21	13.1	14.1	1.076	8	7	15	10
INTEGRAL CALCULUS	16	11.5	11.6	1.009	10	14	7	9
PART A-NO CALCULATOR	28	18.4	18.3	.995	9	11	15	5
PART B-WITH GRAPHING CALCULATOR	17	10.0	10.5	1.05	6	11	13	10
SERIES ITEMS	8	3.7	3.1	.838	16	12	8	4
CALCULATOR ACTIVE ITEMS	5	3.3	3.2	.969	9	12	17	2
Multiple-Choice Summary		**34.1**	**34.5**		**9**	**10**	**15**	**6**

Performance on Free-Response Section (Maximum Possible Score = 54)

					Number of Students in Your Group			
Question/Problem	Max Possible Score	Global Mean	Group Mean		Lowest Fourth	Second Fourth	Third Fourth	Highest Fourth
PARAMETRIC PARTICLE MOTION	9	5.2	4.9	.942	11	13	10	6
MODEL NUMERICALLY/ANALYTICALLY-AB & BC	9	5.5	6.0	1.091	6	11	10	13
PERIMETER/VOLUME	9	5.5	5.6	1.018	12	7	9	12
GRAPHICAL ANALYSIS OF F/FTC/MVT-AB & BC	9	4.1	4.4	1.073	12	6	7	15
MODELING WITH DIFF EQ-AB & BC	9	3.5	3.1	.886	10	13	10	7
TAYLOR SERIES	9	4.0	4.1	1.025	8	8	16	8
Free-Response Summary		**27.9**	**28.1**		**8**	**11**	**12**	**9**

Indicates that the distribution is not displayed because more than half of the total AP global group earned the same score.

Interpreting this report

The following columns appear in each table: Number of Questions (Multiple Choice table only), Max Possible (Free Response table only), Global Mean and Group Mean. The Global Mean column provides all AP students' average scores on specific content areas; the Group Mean column provides this information for your students. The right side of each table, Number of Students in Group, shows the number of your students that fell into the fourths. Fourths are derived from dividing the total student population equally into four parts based on their performance in each content area. The numbers in the Number of Questions column for the multiple-choice content areas are simply the maximum possible scores. Beginning in 2011, no points will be deducted for incorrect answers to multiple-choice questions. The mean score for the content areas will be the average number of multiple choice questions answered correctly.

Some AP Exams have free-response questions that allow students to choose between topics; mean scores are not provided for those questions/problems of the exam. This is because the populations of AP students choosing each question when choice is permitted can be quite different.

CollegeBoard

Gladys Wood

CALCULUS MANUAL

You are going to create a reference manual for the topics we have studied in Calculus with a table of contents and each entry containing the following:

1. On a numbered page, write the topic as a title.
2. **State any relevant definition or theorem.**
3. State the specific problem(s) that necessitate the use of the theorem or procedure under the general topic. (Find your own examples from homework problems – do NOT use problems from the class notes or the book!)
4. Working in two columns on the page, number each step in the symbolic form of the solution in the left-hand column and then write, **in your own words** (use language YOU understand), the procedure used in each step in the right-hand column.
5. The finished product must have a TABLE OF CONTENTS, with page numbers, so I recommend that you maintain it as you create your manual. Remember that the goal is for your finished product to be a "reference book" in times of need in future courses, so create it with this in mind, with the idea that you will bind it in some way.
6. Please note: I may add some entry types during the course of the year.

Calculus Entries:

1. How to solve an absolute value inequality 2 ways ("and" & "or").
2. How to graph the basic transformations of a function.
3. How the graphs of a function for which the limit as $x \to a$ would not exist could look. Use at least three different possibilities.
4. How to prove that a function is continuous at a point.
5. How to prove that a function is continuous on a closed interval. (Include a piecewise function in your examples.)
6. How the graphs of functions that fail each part of the continuity definition could look. Use three examples. (Include Removable and Non-removable Discontinuity)
7. How to implement the Intermediate Value Theorem.
8. How to use the Definition of the Derivative and its Alternate Form to differentiate.
9. How to differentiate polynomial and rational functions, using Basic, Product, Quotient and Chain Rules.
10. How to relate continuity and differentiability.
11. How to find an equation of a line tangent and normal to a graph. (Use the quick way for finding the derivative.)
12. How to implement the Mean Value Theorem for Derivatives. Illustrate graphically.
13. How to use the First Derivative Test to graph a function.
14. How to find local extrema using the Second Derivative Test.
15. How to find absolute extrema on a closed interval.
16. How to differentiate implicitly, with an explanation of the need for this differentiation method.
17. How to solve related rate problems (3 types).
18. How to graph a function of $f(x)$ given the graphs of $f'(x)$ and $f''(x)$.
19. How to graph a function of $f(x)$ given information in a chart about $f'(x)$ and $f''(x)$: $f(x)$ must be two types: 1 polynomial function and 1 rational function.
20. How to solve an optimization problem. (Max./Min.)
21. How to interpret differential equations geometrically using slope fields and their relationship to derivatives of implicitly defined functions.
22. How to compute the Riemann Sum over four equal subintervals for $y = x^2 + 4$ on the interval $[0, 8]$ using left and right endpoints and midpoints.

Name________________________ Date ________________ # ______

Chapter 4 Practice Sheet 1 page 2

(5) If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

(6) If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

(A) $\int_{-1}^{1} f(x)\,dx > 0$

(B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

(C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

(D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

(E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

(7) $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2(\sqrt{2}-1)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$ (D) $2(\sqrt{2}-1)$ (E) $2(\sqrt{2}+1)$

(8) $\int_0^1 x(x^2+2)^2\,dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

(9) What is the average value of y for the part of the curve $y = 3x - x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

Name: Date: Number:

Stuff You Must Know Cold – Algebra 1

Powers

$13^2 =$

$14^2 =$

$15^2 =$

$16^2 =$

$17^2 =$

$18^2 =$

$19^2 =$

$20^2 =$

$21^2 =$

$22^2 =$

$23^2 =$

$24^2 =$

$25^2 =$

$2^3 =$

$3^3 =$

$4^3 =$

$5^3 =$

$6^3 =$

$7^3 =$

$8^3 =$

$9^3 =$

$10^3 =$

$11^3 =$

$12^3 =$

$2^4 =$

$3^4 =$

$4^4 =$

$5^4 =$

Fractions

Integers

Powers

$2^5 =$

$3^5 =$

$4^5 =$

$5^5 =$

$2^6 =$

$2^7 =$

$2^8 =$

$2^9 =$

$2^{10} =$

$2^{11} =$

$2^{12} =$

Factorials

$0! =$

$1! =$

$2! =$

$3! =$

$4! =$

$5! =$

$6! =$

$7! =$

Linear Equations

Parent Function:

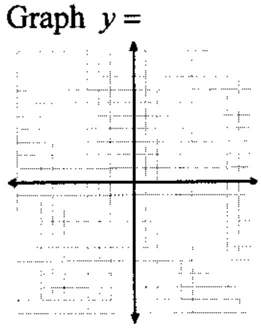
Standard Form:

Slope-Intercept Form:

Point-Slope Form:

Slope:

Graph $y =$



label 3 points

Quadratic Equations

Parent Function:

General Form:

Standard Form:

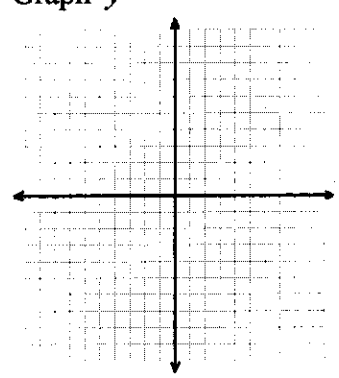
Intercept Form:

Vertex:

Axis of Symmetry:

Quadratic Formula:

Graph $y =$



label 3 points

Inequality Meanings

$<$

$\leq$

$>$

$\geq$

Three ways to solve a system of equations

1.

2.

3.

Name: Date: Number:

Stuff You Must Know Cold – Algebra 1

Order of Operations

1.

2.

3.

4.

Absolute Value

$a \geq 0$

$|a| =$

$|-a| =$

Function Definitions

Domain:

Range:

Function:

Direct Variation:

Indirect Variation:

Roots:

Distance Formula (physics)

Pythagorean Theorem

Properties (use a, b, c)

Commutative
Addition:

Multiplication:

Associative
Addition:

Multiplication:

Distributive:

Measures of Central Tendency

Mean:

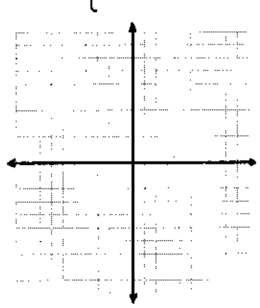
Median:

Mode:

Range:

Piecewise Graph

$f(x) = \{$



label 3 points

Distance Formula (between two points)

$d =$

Midpoint Formula

$(m_1, m_2) =$

Parallel and Perpendicular Lines

If $y = mx + b$

Parallel line slope:

Perpendicular line slope:

Dimensional Analysis

$(°C \Leftrightarrow °F, in \Leftrightarrow cm, ft \Leftrightarrow mi, etc.)$

Convert

Solve The System

x =
y =

Discriminant

Formula:

Nature of roots:

> 0

< 0

$= 0$

Name: Date: Number:

Stuff You Must Know Cold – Geometry

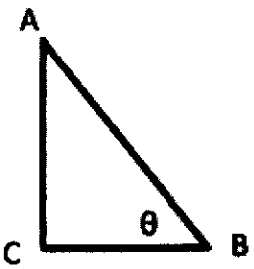
Pythagorean Theorem:

Trigonometry:

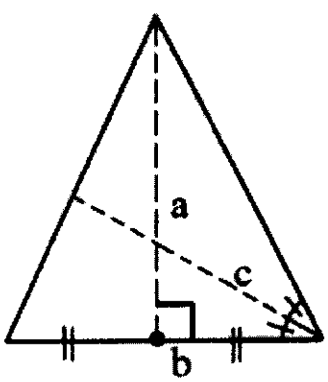
$\sin\theta =$

$\cos\theta =$

$\tan\theta =$



Parts of a Triangle:



a:

b:

c:

Similarity

Ratio of sides a:b

Ratio of perimeters

Ratio of areas

Ratio of volumes

Triangle Congruence:

1.

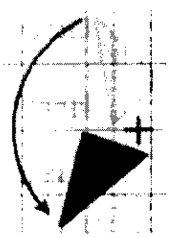
2.

3.

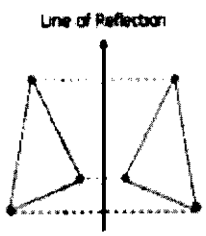
4.

5.

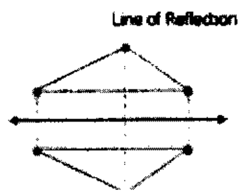
Transformations:



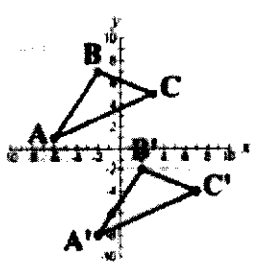
is a ________



______ reflection



________ reflection



is a ___________

Logic

Conditional Statement:

Converse:

Inverse:

Contrapositive:

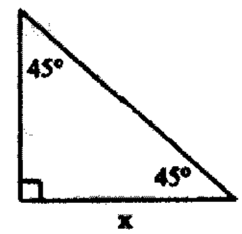
Perimeter Formulas:

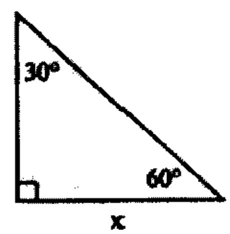
Square:

Rectangle:

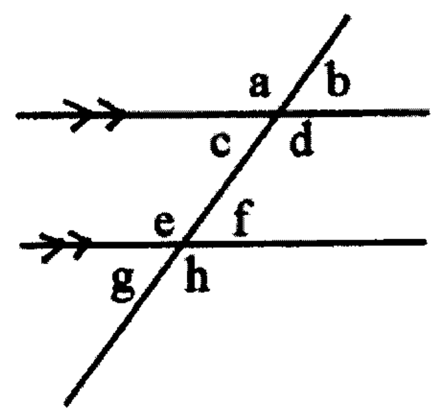
Circumference of a Circle:

Special Right Triangles:





Parallel Lines:



a is congruent to:

a is supplementary to:

Name: Date: Number:

Stuff You Must Know Cold – Geometry

Area Formulas:

Square:

Rectangle:

Parallelogram:

Trapezoid:

Circle:

Right Triangle:

Any Triangle (Heron's Formula):

Equilateral Triangle:

Regular Polygon:

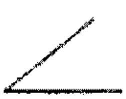
Surface Area Formulas:

Cube:

Sphere:

Cylinder:

Angles:

 is an _______ angle

 is an ________ angle

Complementary angles add up to ____

Supplementary angles add up to ____

Volume Formulas:

Cube:

Prism:

Cylinder:

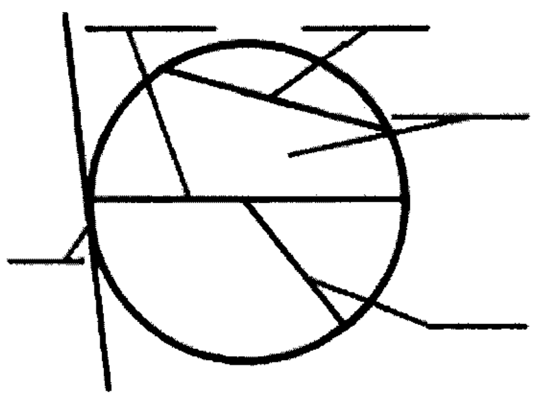
Pyramid:

Cone:

Sphere:

Volume is measured in ____ units

Parts of a Circle:



Polygon Interior Angle Sums:

Triangle:

Quadrilateral:

Regular Polygon:

Arc and Sector

Arc Length:

Sector Area:

Roots to Know:

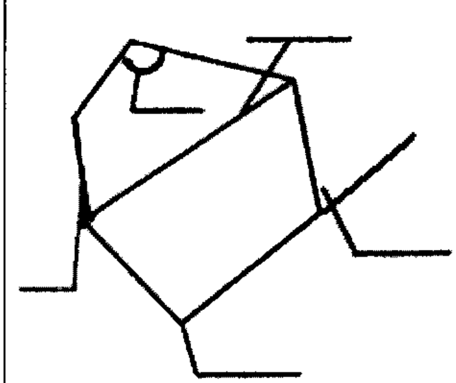
$\sqrt{2} \approx$

$\sqrt{3} \approx$

Polygon Names:

Sides:	Name:
3	
4	
5	
6	
7	
8	
9	
10	

Polygon Parts:



Lines:

 is a __________

 is a __________

 = _________

Y = 2/3x + 4

Give a equation of a line Parallel:

Give a line Perpendicular:

Name: Date: Number:

Stuff You Must Know Cold – Algebra II

Arithmetic Series

$t_n =$

$S_n =$

Geometric Series

$t_n =$

$S_n =$

S

Probability and Statistics

${}_nP_r =$

${}_nC_r = \left(\frac{n}{r}\right) =$

Basic Counting Principle with m,n, and *l* different items =

Transformations (a,b,h,k >0)

$y = f(x) - k$

$y = f(x - h)$

$y = -f(x)$

$y = f(-x)$

$y = f^{-1}(x)$

$y = af(x)$

$y = f(bx)$

Fractions

$\frac{0}{\#} =$

$\frac{0}{0} =$

$\frac{\#}{0} =$

Conjugate
of a + b is ____________

(a-b)(a+b)=

Horizontal Asymptote Rules:

$$y = \frac{ax^m + \ldots}{bx^n + \ldots}$$

1.

2.

3.

Vertical Asymptote Rules:

Imaginary Numbers:

$\sqrt{-1} =$

$i^2 =$

$i^3 =$

$i^4 =$

Arithmetic Operations

$ab + ac =$

$\frac{a}{b} + \frac{c}{d} =$

$\frac{a+b}{c} =$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} =$

$\frac{\left(\frac{a}{b}\right)}{c} =$

$\frac{a}{\left(\frac{b}{c}\right)} =$

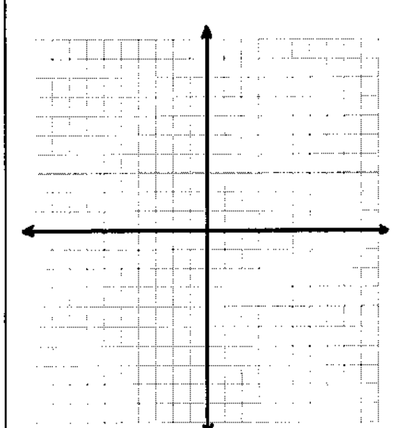
$a\left(\frac{b}{c}\right) =$

$\frac{a-b}{c-d} = \frac{b-a}{\quad}$

$\frac{ab+ac}{a} =$

Parent Functions

Graph
$y =$



Name: Date: Number:

Stuff You Must Know Cold – Algebra II

Exponents and Radicals	**Logarithm Rules**	**Conics** (standard forms)
$a^{0=} \quad ,a \neq 0$	Change from log to exponential $\log_b y = x$	Circle:
$(ab)^n =$	$\ln y = x$	Parabola Vertical:
$a^x a^y =$	$\log y = x$	Horizontal:
$\sqrt{a} =$	Change from exponential to log $b^x = y$	Ellipse Vertical:
$\frac{a^x}{a^y} =$	More log rules $\log_a a =$	Horizontal:
$\sqrt[n]{a} =$	$\log_a 1 =$	Hyperbola Vertical:
$\left(\frac{a}{b}\right)^x =$	$\log_a a^n =$	Horizontal:
$\sqrt[n]{a^m} =$	$\log_b(mn) =$	Complete the Square:
$a^{-x} =$	$\log_b\left(\frac{m}{n}\right) =$	**Factoring**
$\sqrt[n]{ab} =$	$\log_b m^n =$	Difference of Squares $a^2 - b^2 =$
$\left(a^x\right)^y =$	Change of base formula $\log_c a =$	Difference of Cubes $a^3 - b^3 =$
$\sqrt[n]{\frac{a}{b}} =$	**Intercepts**	Sum of Cubes $a^3 + b^3 =$
Interest/Half-Life	To find the x-intercept of any function:	Perfect Square Trinomial $a^2 - 2ab + b^2 =$
Compound Interest:	To find the y-intercept of any function:	$a^2 + 2ab + b^2 =$
Continuously Compounded:		Grouping $ac + ad + bc + bd =$
Exponential Growth/Decay:		

Name: Date: Number:

Stuff You Must Know Cold – Pre-Cal

Even – Odd Functions

If $f(x)$ is even, then

If $f(x)$ is odd, then

Composite Functions

$f(x) =$

$g(x) =$

$f(g(x)) =$

Triangles

Law of Cosines:

Law of Sines:

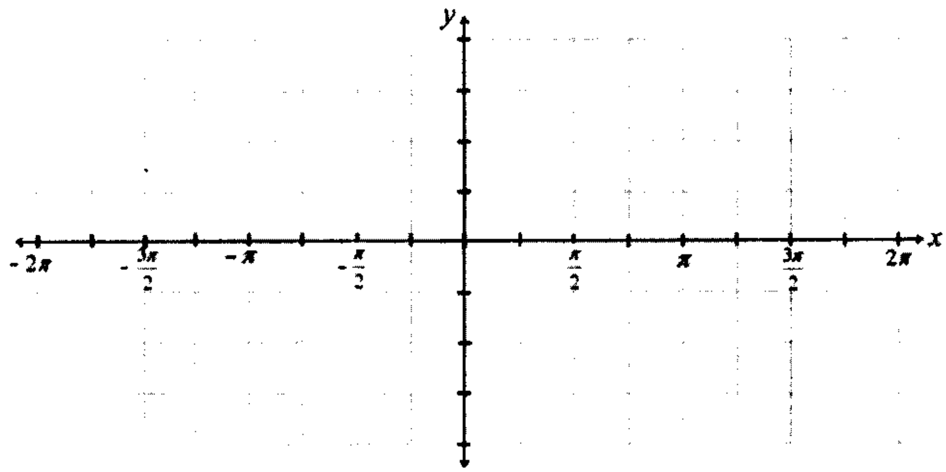
Values of Trigonometric Functions for Common Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°			
$\frac{\pi}{6}$			
$\frac{\pi}{4}$			
$\frac{\pi}{3}$			
$\frac{\pi}{2}$			
π			

Know both the ***inverse trig*** *and the* ***trig*** *values. E.g.* $\tan^{-1}(1)$

Trig Graph

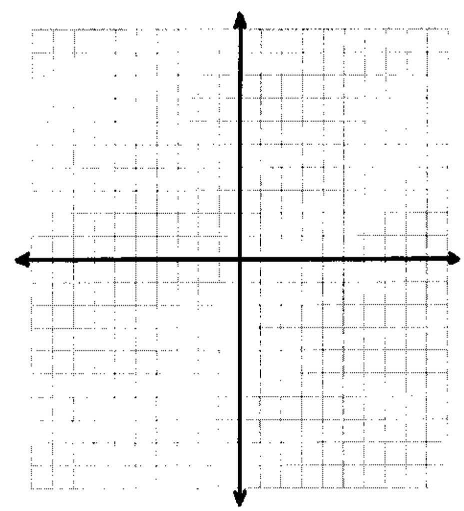
$y =$



Parametric Equations

Graph
$x(t) =$

$y(t) =$



Polar Equations

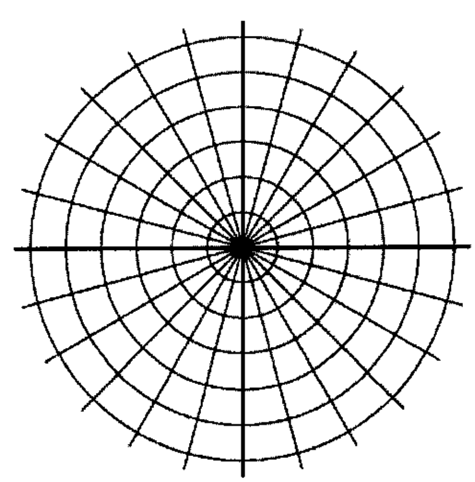
Standard Formulas

$x =$

$y =$

$\tan\theta =$

$x^2 + y^2 =$



Graph
$r =$

Name: Date: Number:

Stuff You Must Know Cold – Pre-Cal

Trig Identities

Double Angle

$\sin(2x) =$

$\cos(2x) =$

$=$

$=$

Power Reduction

$\sin^2 x =$

$\cos^2 x =$

Pythagorean

$\sin^2 x + \cos^2 x =$

$1 + \tan^2 x =$

$\cot^2 x + 1 =$

Reciprocal

$\sec x =$

$\cos x \cdot \sec x =$

$\csc x =$

$\sin x \cdot \csc x =$

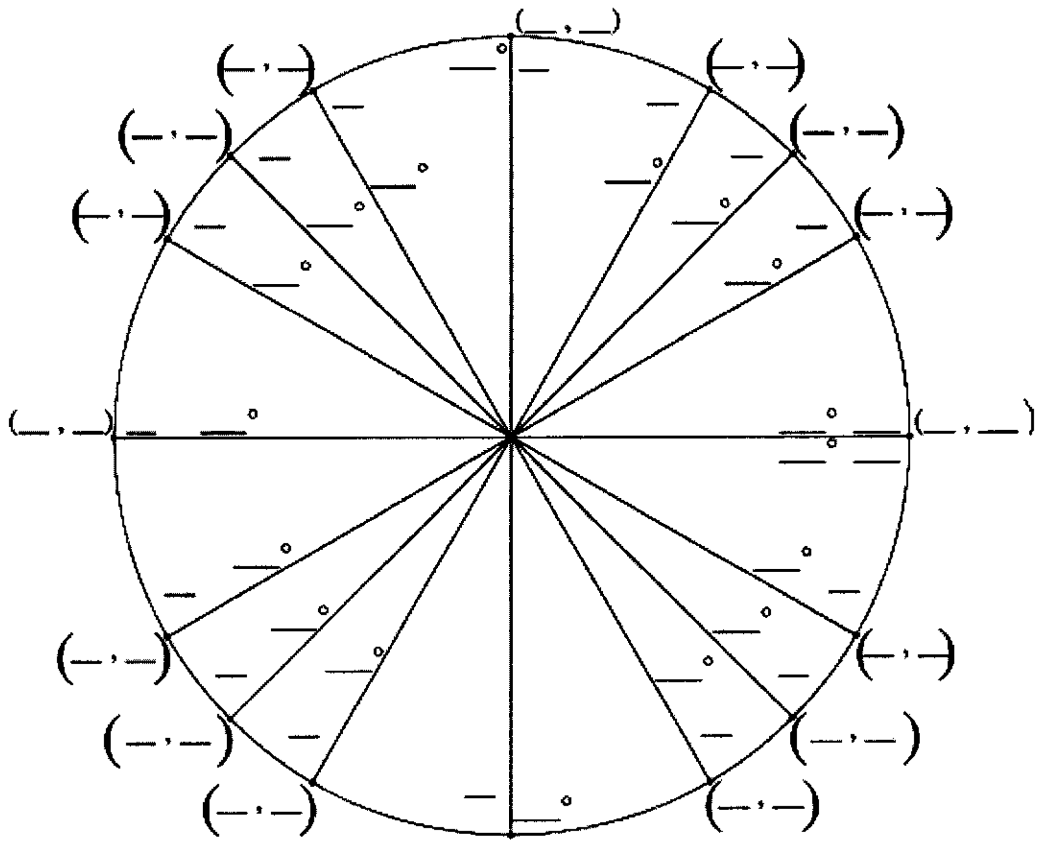
Even/Odd

$\sin(-x) =$

$\cos(-x) =$

$\tan(-x) =$

Trig Unit Circle

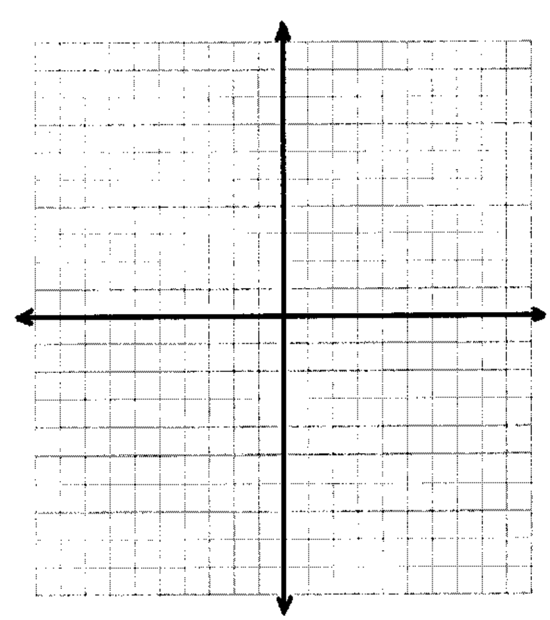


Vectors

$\vec{u} = \langle \quad , \quad \rangle$

$\|\vec{u}\| =$

Graph $\vec{u}$



Name: Date: Number:

Stuff You Must Know Cold – Pre-Cal

Sequences and Series

$\sum_{i=}$

Perfect Squares

$(u+v)^2 =$

$(u-v)^2 =$

Perfect Cubes

$(u+v)^3 =$

$(u-v)^3 =$

Binomial Theorem

$(\quad) =$

$+ - \times \div$ Rational Functions

Matrices

$A = \begin{bmatrix} \quad \end{bmatrix}$

$B = \begin{bmatrix} \quad \end{bmatrix}$

$A - B = \begin{bmatrix} \quad \end{bmatrix}$

$AB = \begin{bmatrix} \quad \end{bmatrix}$

Decomposition of Fractions

$\underline{\qquad\qquad} =$

$\sqrt{x^2} =$

$e \approx$

$\pi \approx$

Synthetic Division

$\underline{\qquad\qquad} =$

$\sin(\alpha \pm \beta) =$

$\cos(\alpha \pm \beta) =$

Projectile Motion

$x(t) =$

$y(t) =$

Name: Date: Number:

Stuff You Must Know Cold – Cal AB/BC

Limits	Curve Sketching and Analysis	More Derivatives
Notation for: Limit from the left of $f(x)$ as $x \to a$ Limit from the right of $f(x)$ as $x \to a$ Theorems: $\lim_{x\to a} f(x) = F$ *and* $\lim_{x\to a} g(x) = G$ $\lim_{x\to a}(f(x)+g(x)) =$ $\lim_{x\to a}(f(x)-g(x)) =$ $\lim_{x\to a}(f(x)\cdot g(x)) =$ $\lim_{x\to a}(f(x))^n =$ $\lim_{x\to a}\frac{f(x)}{g(x)} =$ $\lim_{x\to 0}\frac{\sin x}{x} =$ $\lim_{x\to 0}\frac{1-\cos x}{x} =$ **Definition of Continuity:** A function is continuous at the point x=a if and only if: 1. 2. 3. **Extreme Value Theorem**	Critical Points: Global Min: Global Max: Point of Inflection: **Derivatives** Definition of Derivative $\frac{d}{dx}\left(f(x)\right) =$ Alternate Form of Def. of Derivative $\frac{d}{dx}\left(f(x)\right)$ *at* $x = a$ Chain Rule $\frac{d}{dx}[f(u)] =$ Product Rule $\frac{d}{dx}(uv) =$ Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right) =$ Where u and v are functions of x	Where u is a function of x and a is a constant $\frac{d}{dx}(x^n) =$ $\frac{d}{dx}(\sin u) =$ $\frac{d}{dx}(\cos u) =$ $\frac{d}{dx}(\tan u) =$ $\frac{d}{dx}(\cot u) =$ $\frac{d}{dx}(\sec u) =$ $\frac{d}{dx}(\csc u) =$ $\frac{d}{dx}(\ln u) =$ $\frac{d}{dx}(e^u) =$ $\frac{d}{dx}(\sin^{-1} u) =$ $\frac{d}{dx}(\cos^{-1} u) =$ $\frac{d}{dx}(\tan^{-1} u) =$ $\frac{d}{dx}(\cot^{-1} u) =$ $\frac{d}{dx}(a^u) =$ $\frac{d}{dx}(\log_a u) =$ **Intermediate Value Theorem**

Name: Date: Number:

Stuff You Must Know Cold – Cal AB/BC

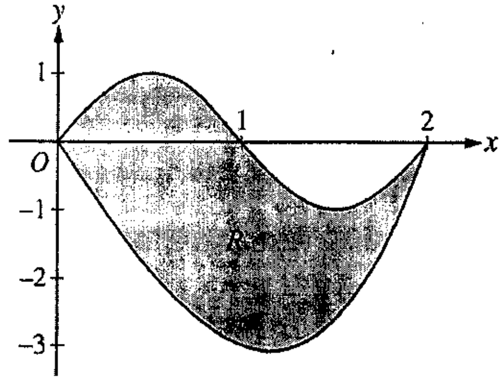
The Mean Value Theorem (derivatives)	Distance, Velocity, and Acceleration	Parametric Equations
Rolle's Theorem	s(t) is the position function, $<x(t), y(t)>$ is the position in parametric	$\frac{dy}{dx} =$
The Fundamental Theorem of Calculus	velocity =	$\frac{d^2y}{dx^2} =$
Corollary to FTC $\frac{d}{dx}\int_a^{g(x)} f(t)dt =$	acceleration =	**Polar Curves**
Area Under The Curve (Trapezoids)	velocity vector =	Area =
Mean Value Theorem for Integrals (Average Value)	acceleration vector =	Slope =
Solids of Revolution and Friends	speed (rectangular and parametric) =	**Taylor Series**
Disk Method	displacement =	**Maclaurin Series**
Washer Method	distance (rectangular and parametric) =	$e^x =$
General volume equation	average velocity =	$\cos x =$
Arc Length (rectangular)	**l'Hôpital's Rule** (Bernoulli's Rule)	$\sin x =$
	Euler's Method	$\frac{1}{1-x} =$
	Integration by Parts	$\ln(x+1) =$
	Logistics $\frac{dP}{dt} =$	**Series Tests/Error Bound**

	AB1	AB2	AB3	AB4	AB5	AB6	BC	BC	BC
2011									
2011B									
2010									
2010B									
2009									
2009B									
2008									
2008B									
2007									
2007B									
2006									
2006B									
2005									
2005B									
2004									
2004B									
2003									
20003B									

Dixie Ross

Name ______________________ Date ______________ #: ______

AB1 2008 GC



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

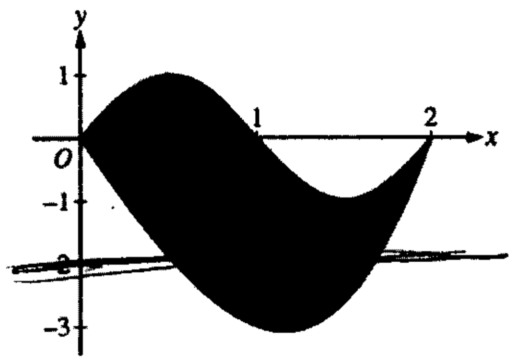
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(a)

$\sin(\pi x) = x^3 - 4x$

$x = 2$

$A = \int_0^2 \sin(\pi x) - (x^3 - 4x)\,dx$

$A = 4$

Work for problem 1(b)

$-2 = x^3 - 4x$

$x = .5391887$, and 1.6751309

$A = \int_{.539187}^{1.6751309} (-2) - (x^3 - 4x)\,dx$

$= \int_{.5391887}^{1.6751309} -2 - x^3 + 4x\,dx$

Continue problem 1 on page 5

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Work for problem 1(c)

$$V = \int_0^2 (\sin(\pi x) - (x^3 - 4x))^2 \, dx$$

$$V = 9.978344126$$

Work for problem 1(d)

$$V = \int_0^2 (\sin(\pi x) - (x^3 - 4x))(3 - x) \, dx$$

$$V = 8.369953106$$

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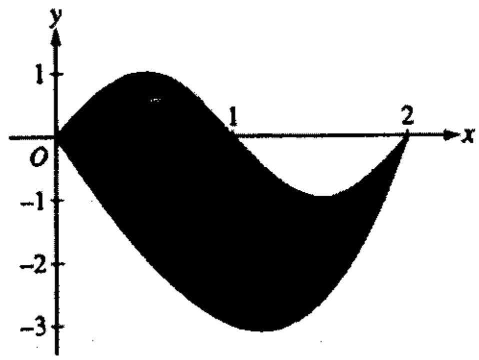
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$\sin(\pi x) = x^3 - 4x$

$x = -2 \quad x = 0 \quad x = 2$

$A = \sin(\pi x) - (x^3 - 4x)$

$\int_0^2 [\sin(\pi x) - (x^3 - 4x)]\,dx = 4$

Work for problem 1(b)

$\sin(\pi x) - (x^3 - 4x)$

$y = -2$

$[\sin(\pi x) - (x^3 - 4x)] - (-2)$

$-x^3 + 4x$

$\int_0^2 ([(\sin(\pi x) - x^3 + 4x)] + 2)\,dx$

Continue problem 1 on page 5.

Work for problem 1(c)

cross section = square

$A = s^2$

$= [\sin \pi x - x^3 + 4x]^2$

$V = \int_0^2 [\sin \pi x - x^3 + 4x]^2 dx = 9.9783$

Work for problem 1(d)

$h(x) = 3 - x$ (depth)

$V = \pi \int_0^2 [\sin \pi x - x^3 + 4x](3-x)\,dx$

26.2950

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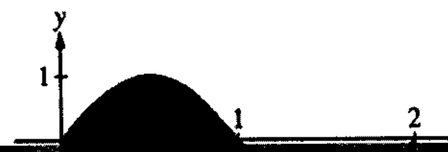
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



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Work for problem 1(c)

$$V_{volume} = \int_0^2 (\sin(\pi x))^2 - (x^3 - 4x)^2 = 8.752$$

Work for problem 1(d)

$$V_{water\ in\ pond} = 4\pi \int_0^2 \frac{1}{2}(3-x)^2 = 17.333\pi$$

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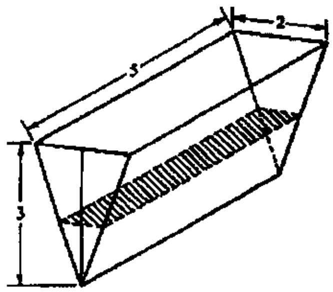
1987 AB5



The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

(b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?

Dixie Ross

RELATED RATES PROJECT

Due:______________________

For this project, you will write an original related rates problem. The problem must be illustrated on a small poster, diorama, or power point. Power points must be printed out before class for another student to work in class. The problem could have a seasonal theme (Halloween, Thanksgiving, Christmas, Hanukkah, Winter Solstice etc. – does not need to be religious in nature). The problem should utilize a mathematical relationship (Pythagorean Theorem, cone, similar triangles, sphere, circle, trig ratios, etc.). Make a 9 point rubric on regular sized paper with the problem correctly worked out. Your problems will be graded on the following criteria:

- 10 points-problem is creative, neat, original and entertaining
- 10 points-problem utilizes correct mathematical relationships
- 10 points-solution is correctly and clearly presented
- 10 points-illustrations (NOT computer generated!)/presentation
- 5 points-working correctly another student's project
- 5 points-difficulty level of the problem

HAVE FUN WITH THIS!! By the way, a well-done problem may appear on the Exam or Final Exam, and its author will receive Bonus Points!!

Name: ______________________ Class: _________________ Date: _________ Number:_______ ID: B

BC Test Up To Definite Integrals (5)

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

a. not defined
b. 4
c. –2
d. 0
e. 2

____ 2. **Find the values of x that give relative extrema for the function $f(x) = 3x^5 - 5x^3$.**

a. None of these
b. Relative maximum: x = -1
Relative minimum: x = 1
c. Relative maximum: 0
Relative minima: x = 1, x = -1
d. Relative maximum: x = 0
Relative minimum: x = $\sqrt{3/3}$
e. Relative maxima: x = 1, x = -1
Relative minimum: 0

____ 3. Which of the following equations expresses y as a function of x?

a. $2x^2y + x = 4y$
b. $3y + 2x - 9 = 17$
c. **Both *a* and *b***
d. **Neither *a* nor *b***

____ 4. **Find $\lim_{x \to -1} \frac{x^2 + 3x + 2}{x^2 + 1}$.**

a. **Does not exist**
b. -1
c. 0
d. ∞

____ 5. Evaluate the integral: $\int x \sec^2 x^2 \, dx$.

a. $\frac{1}{2}x^2 \tan x^2 + C$

b. $\tan x^3 + C$

c. $\frac{1}{2} \tan x^2 + C$

d. $\frac{1}{6}x^3 \sec^3 x^2 + C$

____ 6. Evaluate the integral: $\int (ax + b) \, dx$.

a. $\frac{a}{2}x^2 + bx + C$

b. $\frac{a}{2}x^2 + bx$

c. $\frac{ab}{2}x^2 + C$

d. $a + C$

____ 7. If the function f is defined by $f(x) = x^5 - 1$, then f^{-1}, the inverse function of f is defined by $f^{-1}(x) =$

a. $\sqrt[5]{x} - 1$

b. $\sqrt[5]{x+1}$

c. $\dfrac{1}{\sqrt[5]{x} + 1}$

d. $\dfrac{1}{\sqrt[5]{x+1}}$

e. $\sqrt[5]{x-1}$

____ 8. Evaluate the integral: $\int \sin^3 3x \cos 3x \, dx$.

a. $\frac{1}{12} \sin^4 3x + C$

b. $\frac{1}{8} \sin^4 3x \cos^2 3x + C$

c. $\frac{1}{4} \sin^4 3x + C$

d. $3 \sin^2 3x(3 \cos^2 3x - \sin^2 3x) + C$

____ 9. **The position equation for the movement of a particle is given by $s = (t^2 - 1)^3$ when s is measured in feet and t is measured in seconds. Find the acceleration at two seconds.**

a. 342 units/sec^2
b. 18 units/sec^2
c. 288 units/sec^2
d. 90 units/sec^2

____ 10. **Which of the following functions has a horizontal asymptote at $y = 2$?**

a. $\dfrac{2x^2 - 6x + 1}{1 + x^2}$
b. $\dfrac{2x - 1}{x^2 + 1}$
c. None of these
d. $\dfrac{x - 2}{3x - 5}$
e. $\dfrac{2x}{\sqrt{x - 2}}$

____ 11. **If the point $\left(-3, \frac{1}{2}\right)$ lies on the graph of the equation $2x + ky = -11$, find the value of k.**

a. -34
b. $-\frac{5}{2}$
c. $-\frac{17}{2}$
d. -10

____ 12. The graph of $y = 5x^4 - x^5$ has a point of inflection at

a. $(4,256)$ only
b. $(3,162)$ only
c. $(0,0)$ and $(4,256)$
d. $(0,0)$ only
e. $(0,0)$ and $(3,162)$

____ 13. If $f(x) = 2x^2 + 4$, which of the following will calculate the derivative of $f(x)$?

a. $\dfrac{[2(x+\Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

b. $\lim\limits_{\Delta x \to 0} \dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

c. $\dfrac{(2x^2 + 4 + \Delta x) - (2x^2 + 4)}{\Delta x}$

d. $\lim\limits_{\Delta x \to 0} \dfrac{[2(x+\Delta x)^2 + 4] - (2x^2 + 4)}{\Delta x}$

____ 14. Evaluate the integral: $\int 3\csc^2 x\, dx$.

a. $-3\cot x + C$

b. $6\csc^2 x \cot x + C$

c. $-\frac{1}{3}\csc^3 x + C$

d. $\frac{1}{3}\csc^2 x + C$

____ 15. Find $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4} - 2}{x}$.

a. 0

b. 1

c. ∞

d. $\dfrac{1}{4}$

____ 16. Evaluate the integral: $\int_0^2 |x - 1|\, dx$

a. 0

b. 1/2

c. 1

d. 2

____ 17. Find $\lim\limits_{x \to 2} \dfrac{x-2}{x^2 - 4}$.

a. ∞

b. 1

c. $\dfrac{1}{4}$

d. 0

____ 18. Which of the following defines a function f for which $f(-x) = -f(x)$?

a. $f(x) = \cos x$
b. $f(x) = \log x$
c. $f(x) = e^x$
d. $f(x) = \sin x$
e. $f(x) = x^2$

____ 19. If $f(x) = \dfrac{1}{\sqrt{x}}$ and $g(x) = 1 - x^2$, find $f(g(x))$.

a. $1 - \dfrac{1}{x}$
b. $\dfrac{1}{\sqrt{1 - x^2}}$
c. $\dfrac{1 - x^2}{\sqrt{x}}$
d. $\dfrac{1}{\sqrt{x}} + 1 - x^2$

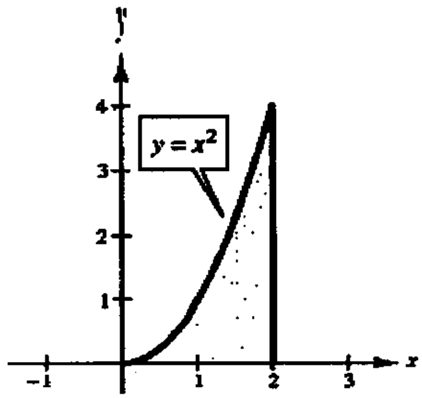
____ 20. Use the properties of sigma notation and the summation formulas to evaluate the given sum: $\sum_{i=1}^{10} (i^2 + 3i - 2)$

a. 126
b. 915
c. 530
d. 128

____ 21. Find all critical numbers for the function $f(x) = \dfrac{x - 1}{x + 3}$.

a. None of these
b. $1, -1$
c. $1, -3$
d. -3
e. 1

____ 22. Which of the following definite integrals represents the area of the shaded region?



a. $\int_0^2 x^2\,dx$

b. $\int_1^2 x^2\,dx$

c. $\int_0^4 x^2\,dx$

d. $\int_0^2 x^2$

____ 23. Find the derivative: $s(t) = \csc \frac{t}{2}$.

a. $\frac{1}{2}\csc\frac{t}{2}\cot\frac{t}{2}$

b. $\frac{1}{2}\cot^2\frac{t}{2}$

c. $-\csc\frac{t}{2}\cot\frac{t}{2}$

d. $-\frac{1}{2}\cot^2\frac{t}{2}$

e. None of these

____ 24. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

a. 1
b. π
c. $\frac{1}{4\pi}$
d. $\frac{1}{\pi}$
e. $\frac{1}{4}$

____ 25. Let $s(n) = \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^2 \left(\frac{2}{n}\right)$. Find the limit of $s(n)$ as $n \to \infty$.

a. $\frac{20}{3}$
b. $\frac{10}{3}$
c. $\frac{17}{12}$
d. $\frac{14}{3}$

____ 26. Evaluate the integral: $\int \sqrt[3]{t}\, dt$.

a. $\frac{3}{2}t^{2/3} + C$
b. $\frac{3}{4}t^{4/3} + C$
c. $\frac{1}{3t^{2/3}} + C$
d. $\sqrt[3]{\frac{1}{2}t^2} + C$

____ 27. Identify the sum that does not equal the others.

a. $\sum_{i=1}^{6}(i + 2)$
b. $\sum_{k=1}^{6}(3k - 2)$
c. $\sum_{j=3}^{8}(3j - 8)$
d. $\sum_{n=0}^{5}(3n + 1)$

____ 28. Use the general power rule to evaluate the integral: $\int x\sqrt{9-5x^2}dx$.

a. $-\frac{4}{15}(9-5x^2)^{3/2}+C$

b. $-\frac{1}{15}(9-5x^2)^{3/2}+C$

c. $\frac{2}{3}(9-5x^2)^{3/2}+C$

d. $-\frac{1}{10}(9-5x^2)^{3/2}+C$

Name:________________________________ Date:____________________ Number:________

Test Up to p-Series A NWNC

1.

A function f has Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x\sin x+3x^2$

(B) $-\cos(x^2)+1$

(C) $-x^2\cos x+x^2$

(D) $x^2e^x-x^3-x^2$

(E) $e^{x^2}-x^2-1$

2. ____________________________

The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0\le t\le 1$, is given by

(A) $\int_0^1\sqrt{t^2+1}\,dt$

(B) $\int_0^1\sqrt{t^2+t}\,dt$

(C) $\int_0^1\sqrt{t^4+t^2}\,dt$

(D) $\frac{1}{2}\int_0^1\sqrt{4+t^4}\,dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

3.

The area of the region enclosed by the graph of $y=x^2+1$ and the line $y=5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

4.

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f ?

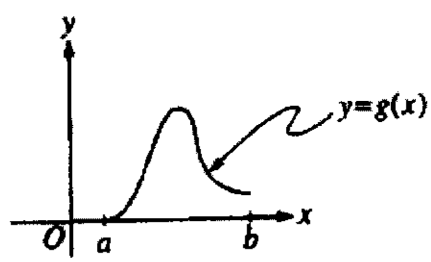
I. $\lim_{x \to 3} f(x)$ exists.

II. f is continuous at $x = 3$.

III. f is differentiable at $x = 3$.

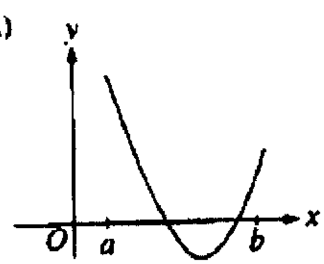
(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

5.

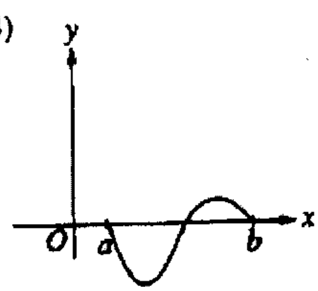


Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a,b]$. Which of the following could be the graph of f on $[a,b]$?

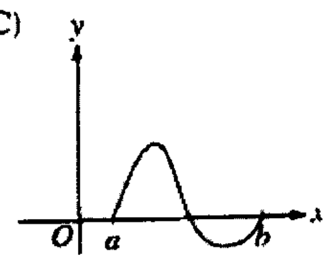
(A)



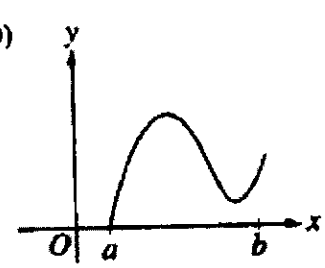
(B)



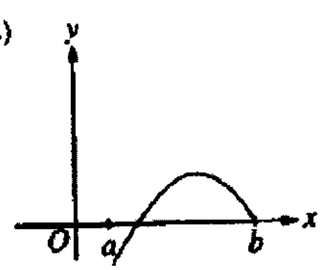
(C)



(D)



(E)



6.

Let f be a function such that $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

7.

If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

8.

Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$

(B) $y = 7x + 7$

(C) $y = 7x + 11$

(D) $y = -5x - 1$

(E) $y = -5x - 5$

9.

If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

10.

If $f(x)=(x-1)^{\frac{3}{2}}+\frac{e^{x-2}}{2}$, then $f'(2)=$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

11.

$\int_1^e \left(\frac{x^2-1}{x}\right)dx=$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

12.

What are all values of x for which the function f defined by $f(x)=x^3+3x^2-9x+7$ is increasing?

(A) $-3<x<1$
(B) $-1<x<1$
(C) $x<-3$ or $x>1$
(D) $x<-1$ or $x>3$
(E) All real numbers

13.

$$\int (3x+1)^5 dx =$$

(A) $\dfrac{(3x+1)^6}{18} + C$

(B) $\dfrac{(3x+1)^6}{6} + C$

(C) $\dfrac{(3x+1)^6}{2} + C$

(D) $\dfrac{\left(\frac{3x^2}{2} + x\right)^6}{2} + C$

(E) $\left(\frac{3x^2}{2} + x\right)^5 + C$

14.

If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

15.

$$\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

16.

$\int x^2 \sin x \, dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$

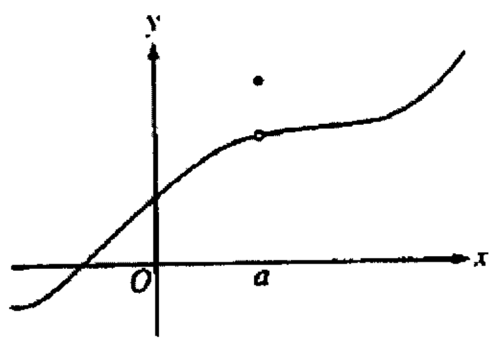
(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$

(C) $-x^2 \cos x + 2x \sin x + 2\cos x + C$

(D) $-\dfrac{x^3}{3} \cos x + C$

(E) $2x \cos x + C$

17.



The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

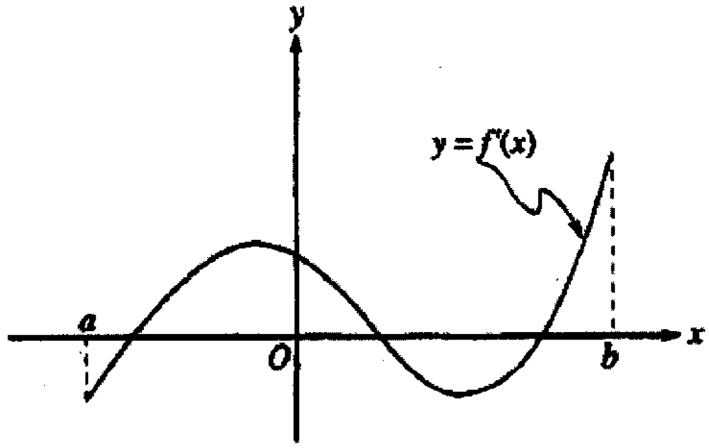
(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x \to a^+} f(x)$ is equal to $\lim_{x \to a^-} f(x)$.

(E) $\lim_{x \to a} f(x)$ exists.

18.



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

19.

The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - 3t^2$ and $y = 2t^3 - 3t^2 - 12t$. For what values of t is the particle at rest?

(A) -1 only (B) 0 only (C) 2 only (D) -1 and 2 only (E) -1, 0, and 2

20.

Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta\, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$ (E) $3\int_0^{\pi} \cos\theta\, d\theta$

21.

A curve C is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$?

(A) $x = -3$

(B) $x = 2$

(C) $y = 8$

(D) $y = -\frac{27}{10}(x + 3) + 8$

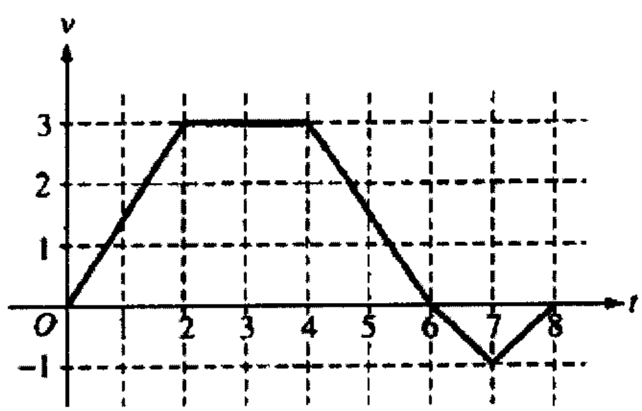
(E) $y = 12(x + 3) + 8$

22.

Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

23.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

24.

x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

25.

If $y = \left(x^3+1\right)^2$, then $\frac{dy}{dx} =$

(A) $\left(3x^2\right)^2$ (B) $2\left(x^3+1\right)$ (C) $2\left(3x^2+1\right)$ (D) $3x^2\left(x^3+1\right)$ (E) $6x^2\left(x^3+1\right)$

26.

If $y = \sin(3x)$, then $\frac{dy}{dx} =$

(A) $-3\cos(3x)$ (B) $-\cos(3x)$ (C) $-\frac{1}{3}\cos(3x)$ (D) $\cos(3x)$ (E) $3\cos(3x)$

27.

The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2,-1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

28.

Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6

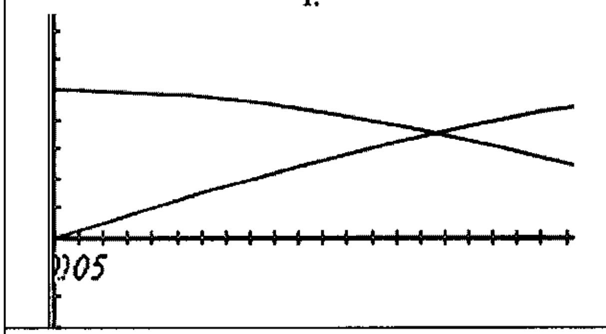
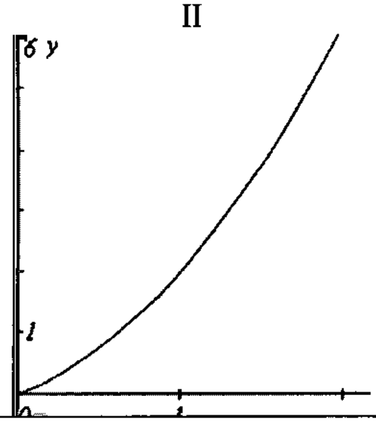
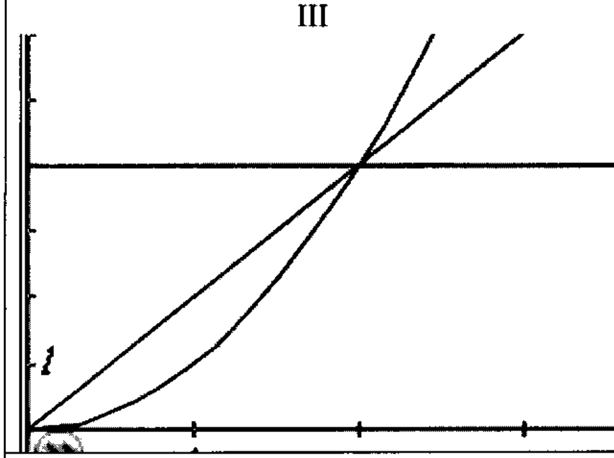
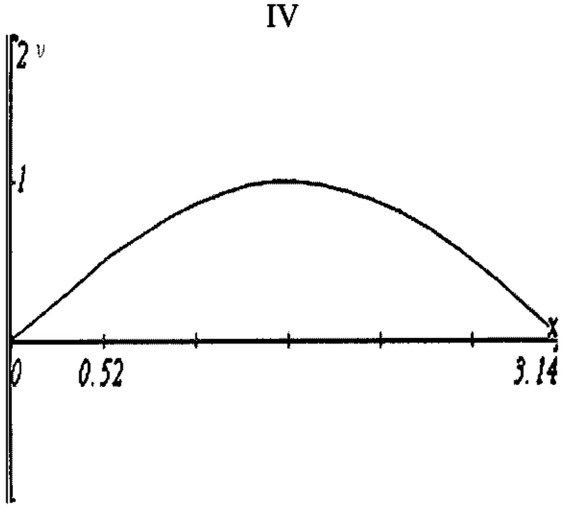
(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

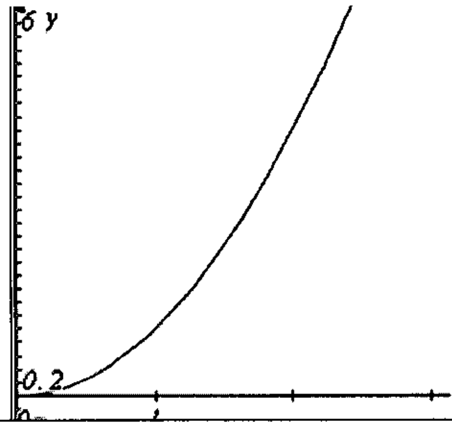
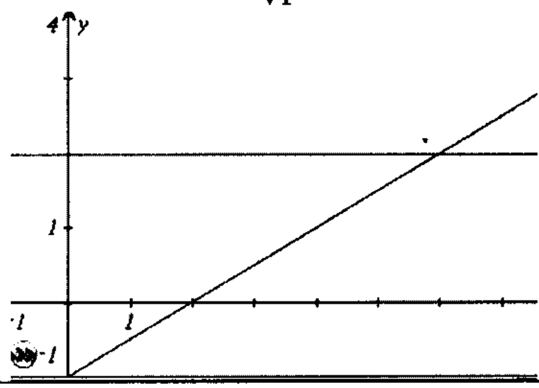
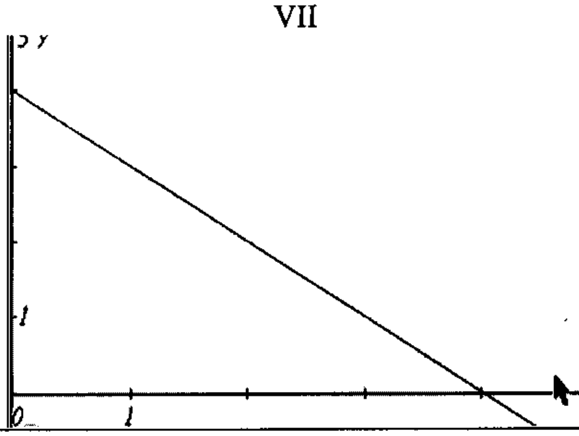
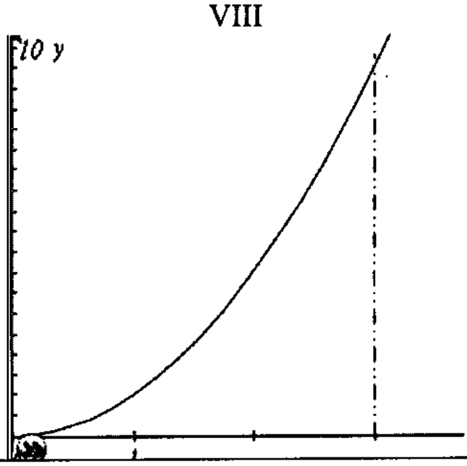
(D) $f(c) = 1$ for at least one c between -3 and 6

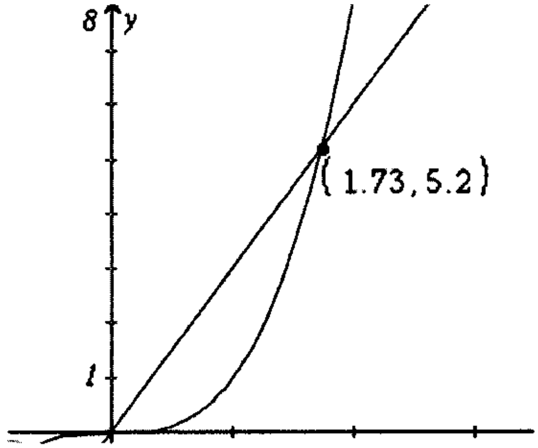
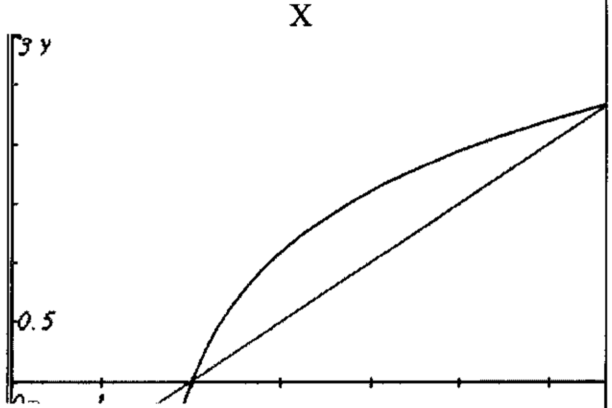
(E) $f(c) = 0$ for at least one c between -1 and 3

Answer Sheet

GRAPH	INTEGRAL	DESCRIPTION
I		
II		
III		
IV		
V		
VI		
VII		
VIII		
IX		
X		

I. 	A $$\pi \cdot \int_0^2 (x^2)^2 \, dx$$	1 Washer; Revolved around the line y=4.
II 	B $$\pi \cdot \int_0^{\pi} (\sin(x))^2 \, dx$$	2 Washer; Revolved around line x=4
III 	C $$\pi \cdot \int_0^9 \left(3-\sqrt{y}\right)^2 \, dy$$	3 Washer; translation of lnx graph revolved around x-axis
IV 	D $$\pi \cdot \int_0^4 \left(4^2-(4-y)^2\right) dy$$	4 Washer; trig graph(s) revolved around x-axis

V  	E $\pi \cdot \int_0^{\sqrt{3}} \left((3\cdot x)^2 - (x^3)^2\right) dx$	5 Disk; Revolved around line x=3
VI  	F $\pi \cdot \int_0^{2} \left((4-x^2)^2 - (4-2\cdot x)^2\right) dx$	6 Washer; revolved around x-axis, involves cubic function
VII  	G $\pi \cdot \int_0^{2} (2\cdot y+2)^2 \, dy$	7 Disk, Volume of a parent function, revolved around x-axis.
VIII  	H $\pi \cdot \int_2^{\frac{20}{3}} \left((\ln(2\cdot x-3))^2 - (0.5\cdot x-1)^2\right) dx$	8 Disk; translation of x^2, revolved around x-axis

IX 	I $$\pi \cdot \int_0^2 \left(x^2+x\right)^2 \, dx$$	9 Disk; revolved around y-axis
X 	J $$\pi \cdot \int_0^{\frac{\pi}{4}} \left((\cos(x))^2-(\sin(x))^2\right) dx$$	10 Disk; trig graph(s) revolved around x-axis

Answer Key

GRAPH	INTEGRAL	DESCRIPTION
I	J	4
II	A	7
III	F	1
IV	B	10
V	I	8
VI	G	9
VII	D	2
VIII	C	5
IX	E	6
X	H	3

B. Definition of Derivative

What you are finding: The derivative of a function is a formula for the slope of the tangent line to the graph of that function. There are two definitions that are commonly used that students should know.

How to find it: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or $f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$. Students need to know that differentiable functions at a point (derivative exists at the point) are necessarily continuous but continuous functions are not necessarily differentiable. These types of problems usually necessitate that students recognize these limits as a derivative and use derivative rules to calculate it.

7. $\lim_{h \to 0} \frac{\sin(\pi + h) - \sin\pi}{h} =$

A. 0　　B. $\cos x$　　C. -1　　D. π　　E. 1

---

8. Let f be a function such that $\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = 2$. Which of the following must be true?

I. f is continuous at $x = 4$.

II. f is differentiable at $x = 4$.

III. The derivative of f' is continuous at $x = 4$.

A. I only　　B. II only　　C. I and II only　　D. I and III only　　E. I, II and III

9. If $f(x) = \tan^{-1}(2x)$, find $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

A. $\dfrac{2}{17}$ B. $\dfrac{1}{17}$ C. $\dfrac{-2}{15}$ D. $\dfrac{-1}{15}$ E. nonexistent

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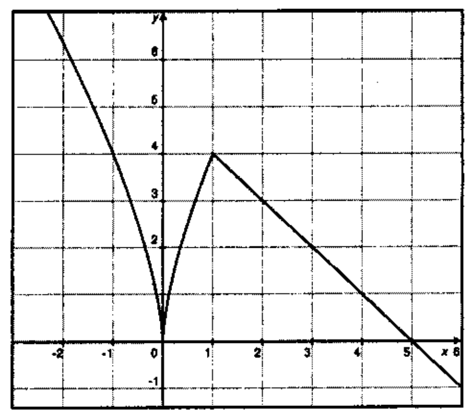
10. The graph of $f(x)$ consists of a curve that is symmetric to the y-axis on [-1, 1] and a line segment as shown to the right. Which of the following statements about f is false?



A. $\lim\limits_{x \to 0} [f(x) - f(0)] = 0$ B. $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x} = 0$

C. $\lim\limits_{h \to 0} \dfrac{f(0+h) - f(0-h)}{2h} = 0$ D. $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2} = -1$

E. $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$ does not exist

F. Continuity and Differentiability

What you are finding: Typical problems ask students to determine whether a function is continuous and/or differentiable at a point. Most functions that are given are continuous in their domain, and functions that are not continuous are not differentiable. So functions given usually tend to be piecewise and the question is whether the function is continuous and also differentiable at the x-value where the function changes from one piece to the other.

How to find it: Continuity: I like to think of continuity as being able to draw the function without picking your pencil up from the paper. But to prove continuity at $x = c$, you have to show that $\lim_{x \to c} f(x) = f(c)$. Usually you will have to show that $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$.

Differentiability: I like to think of differentiability as "smooth." At the value c, where the piecewise function changes, the transition from one curve to another must be a smooth one. Sharp corners (like an absolute value curve) or cusp points mean the function is not differentiable there. The test for differentiability at $x = c$ is to show that $\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x)$. So if you are given a piecewise function, check first for continuity at $x = c$, and if it is continuous, take the derivative of each piece, and check that the derivative is continuous at $x = c$. Lines, polynomials, exponentials and sine and cosines curves are differentiable everywhere.

Example 17: Let $f(x) = \begin{cases} -6x^2 + 14x - 8, x \le 1 \\ \sin(2x - 2), x > 1 \end{cases}$. Show that $f(x)$ is differentiable.

Example 18: Find the values of a and b that make the following function differentiable:

$$y = \begin{cases} ax^3 - 2, x \le 3 \\ b(x-2)^2 + 10, x > 3 \end{cases}$$