Houston Area Calculus Teachers

# AP Reader Discusses the 2014 AB Free Response Questions

November 15, 2014

Cedric French

nfrench@houstonisd.org

## 2014 Mean Scores

**AB scores**

| With zeros and dashes | Without zeros and dashes |
|---|---|
| 1) 2.41 | 1) 3.32 |
| 2) 3.39 | 2) 4.39 |
| 3) 3.29 | 3) 4.20 |
| 4) 2.64 | 4) 3.31 |
| 5) 3.62 | 5) 5.01 |
| 6) 3.92 | 6) 4.82 |

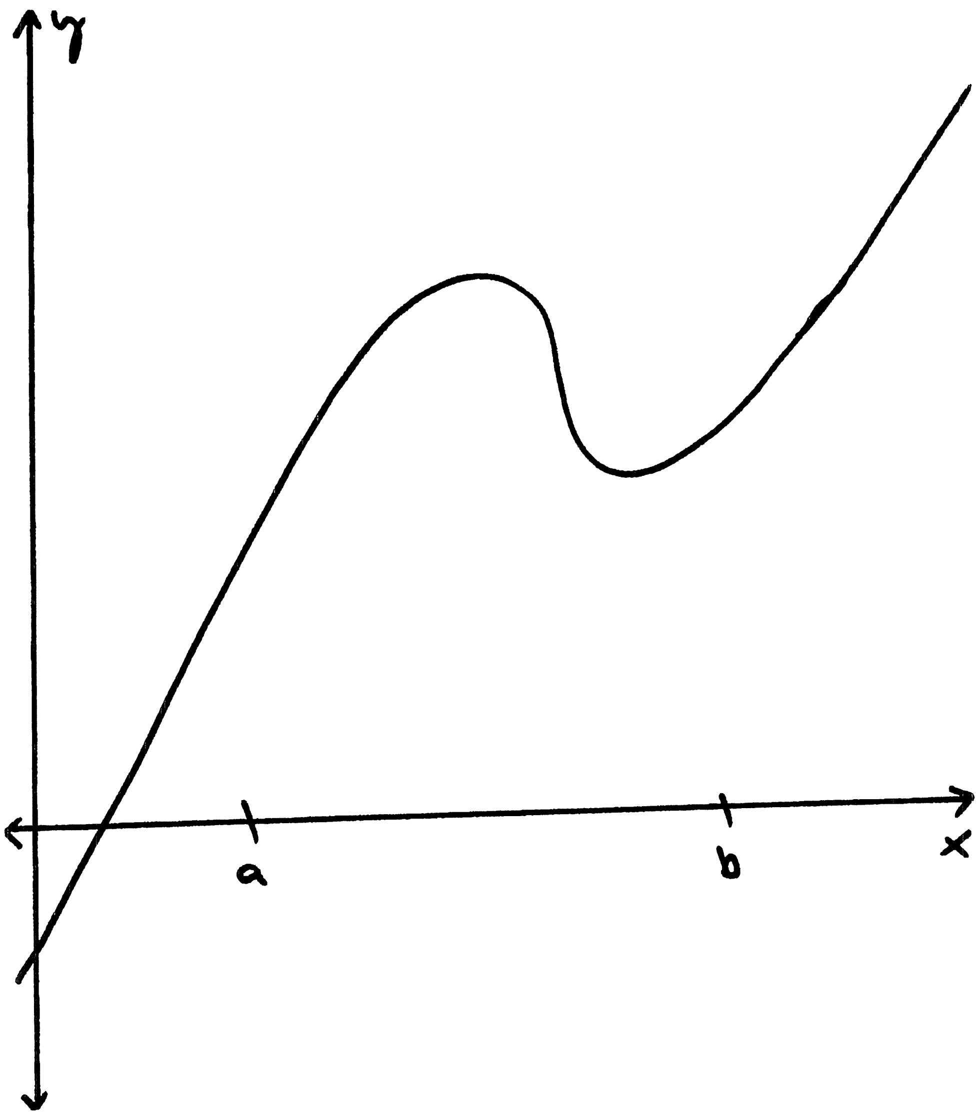
**BC scores**

| With zeros and dashes | Without zeros and dashes |
|---|---|
| 1) 4.27 | 1) 4.68 |
| 2) 3.96 | 2) 4.74 |
| 3) 5.19 | 3) 5.61 |
| 4) 3.97 | 4) 4.25 |
| 5) 4.45 | 5) 4.89 |
| 6) 3.10 | 6) 3.98 |

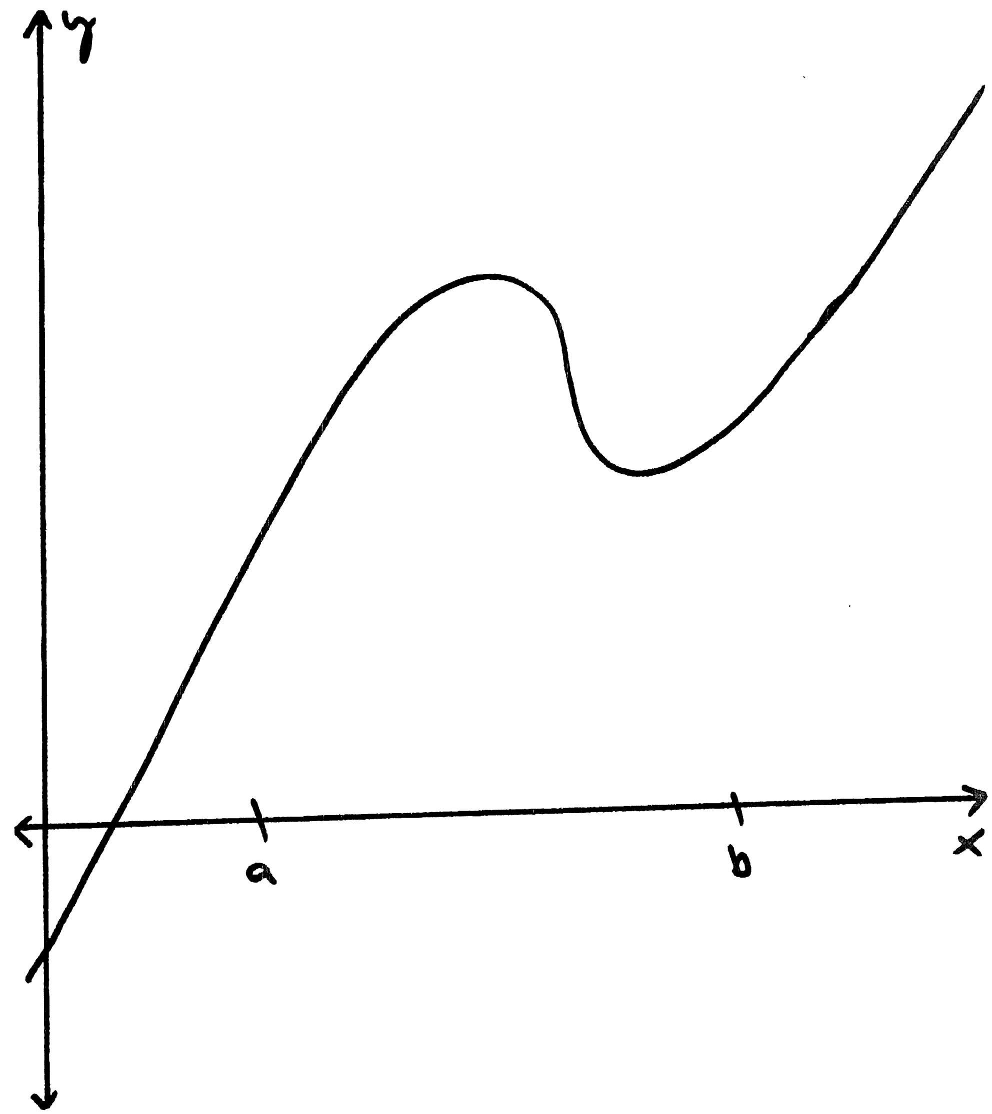
There were:

294,706 AB exams<br>
+ 112,518 BC exams<br>
407,224 Total which were graded by 896 Readers

# Average Value




# Average Value



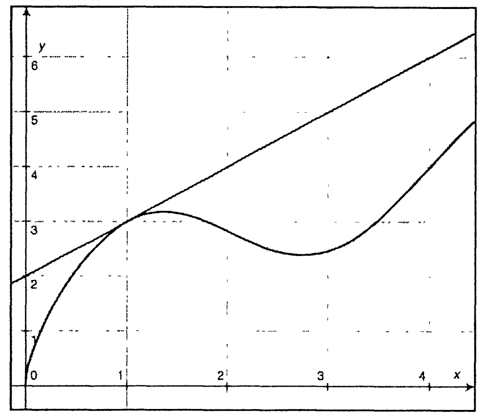
## D. Tangent Lines and Local Linear Approximations

**What you are finding:** You typically have a function $f$ and you are given a point on the function. You want to find the equation of the tangent line to the curve at that point.

**How to find it:** You use your point-slope equation: $y - y_1 = m(x - x_1)$ where $m$ is the slope and $(x_1, y_1)$ is the point. Typically, to find the slope, you take the derivative of the function at the specified point: $f'(x_1)$.

**What you are finding:** You typically have a function $f$ given as a set of points as well as the derivative of the function at those $x$-values. You want to find the equation of the tangent line to the curve at a value $c$ close to one of the given $x$-values. You will use that equation to approximate the $y$-value at $c$. This uses the concept of local linearity – the closer you get to a point on a curve, the more the curve looks like a line.

**How to find it:** You use your point-slope equation: $y - y_1 = m(x - x_1)$ where $m$ is the slope and $(x_1, y_1)$ is the point closest to $c$. Typically, to find the slope, you take the derivative $f'(x_1)$ of the function at the closest $x$-value given. You then plug $c$ into the equation of the line. Realize that it is an approximation of the corresponding $y$-value. If 2nd derivative values are given as well, it is possible to determine whether the approximated $y$-value is above or below the actual $y$-value by looking at concavity. For instance, if we wanted to approximate $f(1.1)$ for the curve in the graph to the right, we could find $f'(x)$, use it to determine the equation of the tangent line to the curve at $x = 1$ and then plug 1.1 into that linear equation. If information were given that the curve was concave down, we would know that the estimation over-approximated the actual $y$-value.




17. If $f(x) = \sin^3(4x)$, find $f'\left(\frac{\pi}{3}\right)$.

A. $-\frac{3}{2}$ B. $-\frac{9}{2}$ C. $-3\sqrt{3}$ D. $3\sqrt{3}$ E. $-\frac{9}{8}$

18. If $f(x) = 2^{x^2 - x - 1}$, find the equation of the tangent line to $f$ at $x = -1$.

A. $y = 2x + 4$ B. $y = -6x - 4$ C. $y = 2 - 2\ln 2(x + 1)$
D. $y = 2 - 6\ln 2(x + 1)$ E. $y = 2 - 3\ln 2(x + 1)$

19. What is the slope of the line tangent to the graph of $y = \dfrac{1}{e^{2x}(x-2)}$ at $x = 1$ ?

A. $\dfrac{-2}{e}$  B. $\dfrac{-1}{e^2}$  C. 0  D. $\dfrac{1}{e^2}$  E. $\dfrac{-2}{e^2}$

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20. What is the equation of the line normal to the graph of $y = 2\sin x \cos x - \sin x$ at $x = \dfrac{3\pi}{2}$?

A. $y = 2x - 3\pi + 1$  B. $y = -2x + 3\pi + 1$  C. $y = \dfrac{x}{2} - \dfrac{3\pi}{4} + 1$

D. $y = -\dfrac{x}{2} + \dfrac{3\pi}{4} + 1$  E. $y = -\dfrac{x}{2} + \dfrac{3\pi}{4} - 1$

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21. **(Calc)** Let $f$ be the function given by $f(x) = \dfrac{4x^2}{e^x - e^{-x}}$. For what positive value(s) of $c$ is $f'(c) = 1$ ?

A. 1.122  B. 0.824 and 4.306  C. 0.258 and 3.260

D. 3.264  E. 0.523 and 4.307

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22. The function $f$ is twice differentiable with $f(-3) = -2$ and $f'(-3) = -4$. For what value of $c$ is the approximation of $f(c)$ using the tangent line of $f$ at $x = c$ equal to $c$?

A. –4.667  B. –0.4  C. –2.8  D. –2.5  E. –3.333

NAME_______________________________

# PROJECT: VOLUMES BY SLICING

We have just completed the numerical computation of the volume of solids with a known cross-section. Your assignment is to make a model of one. You may consult the examples in your notes.

You have been assigned **PROBLEM #**_____________. ***This sheet must accompany your project.***

**Point distribution will be allotted as follows. This project will be counted as a test grade. It will be due_______________. The solid must be on a base which is <u>no larger than 6"x6"</u>. Your solid must have <u>at least 20 cross-sections</u>. You must also completely and correctly work out the numerical volume of your solid** *(That is, you must set up the integral correctly and work it out thoroughly)*. **Turn in rubric along with exact and approximate volumes worked on paper with project.**

| | | PTS | Points Expected | POINTS RECEIVED |
|---|---|---|---|---|
| **ORIGINALITY:** | | ***20 pts.*** | | |
| | Creative Theme | 10 | ________ | ________ |
| | Presentation | 10 | ________ | ________ |
| **APPEARANCE:** | | ***40 pts.*** | | |
| **BASE of Solid:** | | **10 pts.** | | |
| | Accurate Shape of Base | 5 | ________ | ________ |
| | Correct scale marked | 5 | ________ | ________ |
| **CROSS-SECTIONS:** | | **30 PTS** | | |
| | Secure | 5 | ________ | ________ |
| | Accurate Shape | 10 | ________ | ________ |
| | Alternating Color | 5 | ________ | ________ |
| | Completeness of the shape of the solid | 10 | ________ | ________ |
| **EXTENSION:** | | ***40 pts.*** | | |
| | Correct adding the volumes on paper | 15 | ________ | ________ |
| | Correct volume using calculus on paper | 15 | ________ | ________ |
| | Add volumes of cross-sections and compare it to actual volume, % error | 10 | ________ | ________ |
| **TOTAL:** | | | ________ | ________ |

# SOLID VARIETIES

## Cross-Sections are $\perp$ to the BASES BOUNDED BY:

**Type I:**

1. $y = x + 1$ and $y = x^2 - 1$, cross-sections are squares, $\perp$ to $x - axis$.
2. $y = x + 1$ and $y = x^2 - 1$, cross-sections are equilateral triangles, $\perp$ to $x - axis$. (See front cover of your book for this formula.)
3. $y = x + 1$ and $y = x^2 - 1$, cross-sections are rectangles of height 1, $\perp$ to $x - axis$.
4. $y = x + 1$ and $y = x^2 - 1$, cross-sections are semi-ellipses of height 2, $\perp$ to $x - axis$. (See front cover of your book for this formula.)

**Type II:**

5. $y = x^3$, $y = 0$ and $x = 1$, cross-sections are equilateral triangles, $\perp$ to $y - axis$.
6. $y = x^3$, $y = 0$ and $x = 1$, cross-sections are squares, $\perp$ to $x - axis$.
7. $y = x^3$, $y = 0$, and $x = 1$, cross-sections are trapezoids for which $h = b_1 = \frac{1}{2}b_2$ where $b_1$ and $b_2$ are upper and lower bases, $\perp$ to $y - axis$.
8. $y = x^3$, $y = 0$, and $x = 1$, cross-sections are semi-circles, $\perp$ to $y - axis$.
9. $y = x^3$, $y = 0$, and $x = 1$, cross-sections are semi-ellipses whose heights are twice the lengths of their bases, $\perp$ to $y - axis$. (See front cover of your book for this formula.)

**Type III:**

10. $x = y^2$ and $x = 9$, cross-sections are squares, $\perp$ to $x - axis$.
11. $x = y^2$ and $x = 9$, cross-sections are quarter-circles, $\perp$ to $x - axis$.
12. $x = y^2$ and $x = 9$, cross-sections are rectangles of height 2, $\perp$ to $x - axis$.
13. $x = y^2$ and $x = 9$, cross-sections are equilateral triangles, $\perp$ to $x - axis$.
14. $x = y^2$ and $x = 9$, cross-sections are triangles with $h = \frac{1}{4}b$, $\perp$ to $x - axis$.
15. $x = y^2$ and $x = 9$, cross-sections are trapezoids with lower base in $xy -$ plane, upper base $= \frac{1}{2}$ lower base, $h = \frac{1}{4}$ lower base, $\perp$ to $x - axis$.
16. $x = y^2$ and $x = 9$, cross-sections are semi-circles, $\perp$ to $x - axis$.

**Type IV:**

17. circle, $x^2 + y^2 = 4$, cross-sections are isosceles triangles with $h = b$, (triangle base is in the $xy -$ plane), $\perp$ to $x - axis$.
18. circle, $x^2 + y^2 = 4$, cross-sections are semi-circles, $\perp$ to $x - axis$.
19. circle, $x^2 + y^2 = 4$, cross-sections are squares, $\perp$ to $x - axis$.
20. circle, $x^2 + y^2 = 4$, cross-sections are equilateral triangles, $\perp$ to $x - axis$.
21. circle, $x^2 + y^2 = 4$, cross-sections are isosceles right triangles, (right angle formed at the $xy -$ plane), $\perp$ to $x - axis$.

**Type V:**

22. $x = y^2$ and $x = 3 - 2y^2$, cross-sections are rectangles of height 2, $\perp$ to $x - axis$.
23. $x = y^2$ and $x = 3 - 2y^2$, cross-sections are equilateral triangles, $\perp$ to $x - axis$.

Name ______________ Date __________ # ______

## V. Area/Volume Problems

**What you are finding:** Typically, these are problems with which students feel more comfortable because they are told exactly what to do or to find. Area and volume are lumped together because, almost always, they are both tested within the confines of a single A.P. free response question. Usually, but not always, they are on the calculator section of the free-response section.

Area problems usually involve finding the area of a region under a curve or the area between two curves between two values of $x$. Volume problems usually involve finding the volume of a solid when rotating a curve about a line.

**How to find it: Area:** Given two curves $f(x)$ and $g(x)$ with $f(x) \ge g(x)$ on an interval $[a, b]$, the area between $f$ and $g$ on $[a, b]$ is given by $A = \int_{x=a}^{x=b} [f(x) - g(x)]\,dx$. While integration is usually done with respect to the $x$-axis, these problems sometimes show up in terms of $y$: $A = \int_{y=c}^{y=d} [m(y) - n(y)]\,dy$.

**Volume:** Disks and Washers: The method I recommend is to establish the outside Radius $R$, the distance from the line of rotation to the outside curve, and, if it exists, the inside radius $r$, the distance from the line of rotation to the inside curve. The formula when rotating these curves about a line on an interval is given by:

$$V = \pi \int_{x=a}^{x=b} \left( [R(x)]^2 - [r(x)]^2 \right) dx \quad \text{or} \quad V = \pi \int_{y=c}^{y=d} \left( [R(y)]^2 - [r(y)]^2 \right) dy$$

A favorite type of problem is creating a solid with the region $R$ being the base of the solid. Cross sections perpendicular to an axis are typically squares, equilateral triangles, right triangles, or semi-circles. Rather than give formulas for this, it is suggested that you draw the figure, establish its area in terms of $x$ or $y$, and integrate that expression on the given interval.

122. What is the area of the region enclosed by the graphs of $y = x - 4x^2$ and $y = -7x$ ?

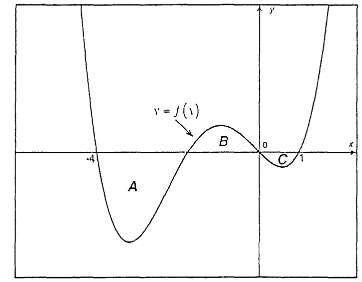
A. $\frac{4}{3}$ B. $\frac{16}{3}$ C. 8 D. $\frac{68}{3}$ E. $\frac{80}{3}$

123. **(Calc)** What is the area of the region in the first quadrant enclosed by the graph of $y = 2\cos x, y = x$, and the $x$-axis?

A. 0.816 B. 1.184 C. 1.529 D. 1.794 E. 1.999

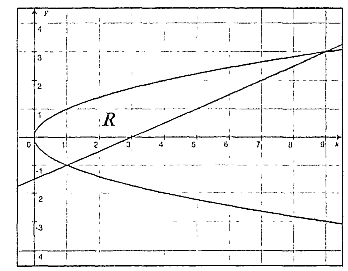


124. The three regions $A$, $B$, and $C$ in the figure to the right are bounded by the graph of the function $f$ and the $x$-axis. If the areas of $A$, $B$, and $C$ are 5, 2, and 1 respectively, what is the value of $\int_{-4}^{1} [f(x) + 2]\,dx$ ?




A. 28 B. 10
C. 6 D. -4
E. -12

125. Region $R$, enclosed by the graphs of $y = \pm\sqrt{x}$ and $y = \frac{x-3}{2}$ is shown in the figure to the right. Which of the following calculations would accurately compute the area of region $R$?



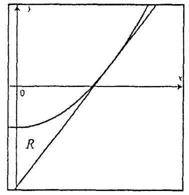

I. $\int_0^9 \left( \sqrt{x} - \frac{x-3}{2} \right) dx$

II. $\int_0^1 2\sqrt{x}\,dx + \int_1^9 \left( \sqrt{x} - \frac{x-3}{2} \right) dx$

III. $\int_{-1}^{3} \left( 2x + 3 - x^2 \right) dx$

A. I only B. II only C. III only D. II and III E. I, II, and III

126. **(Calc)** On the graph to the right, the line $y = kx - 10$, $k$ a constant, is tangent to the graph of $y = x^2 - 4$. Region $R$ is enclosed by the graphs of $y = kx - 10$, $y = x^2 - 4$, and the $y$-axis. Find the area of region $R$.




A. 4.287 B. 4.899 C. 6.124

D. 9.798 E. 12.247

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127. If the region bounded by the $y$-axis, the line $y = 4$, and the curve $y = \sqrt{x}$ is revolved about the $x$-axis, the volume of the solid generated would be

A. $\dfrac{88\pi}{3}$ B. $56\pi$ C. $128\pi$ D. $\dfrac{128\pi}{3}$ E. $\dfrac{640\pi}{3}$

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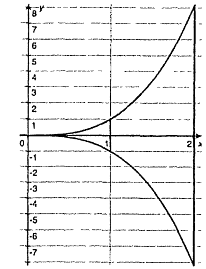
128. **(Calc)** If the region bounded by the $x$-axis, the line $x = \dfrac{\pi}{2}$, and the curve $y = \sin x$ is revolved about the line $x = \dfrac{\pi}{2}$, the volume of the solid generated would be

A. 1.142 B. 1.468 C. 1.793 D. 3.142 E. 3.586



129. A piece of candy is determined by the curves $y = \pm x^3$, for $1 \le x \le 2$ as shown in the figure to the right. For this piece of candy, the cross sections perpendicular to the $x$-axis are equilateral triangles. What is the volume of the piece of candy in cubic units?



A. $\dfrac{127}{7}$ B. $\dfrac{15\sqrt{3}}{2}$ C. $\dfrac{254\sqrt{3}}{7}$

D. $\dfrac{127\sqrt{3}}{21}$ E. $\dfrac{127\sqrt{3}}{7}$

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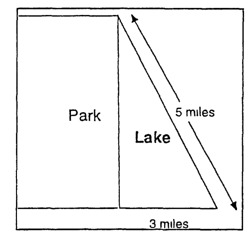
130. A lake shaped in a right triangle is located next to a park as shown in the figure to the right. The depth of the lake at any point along a strip $x$ miles from the park's edge is $f(x)$ feet. Which of the following expressions gives the total volume of the lake?




A. $\int_0^3 \left(4 - \frac{4}{3}x\right) f(x)\,dx$ B. $\int_0^4 \left(3 - \frac{3}{4}x\right) f(x)\,dx$ C. $2\int_0^3 f(x)\,dx$

D. $\int_0^4 f(x)\,dx$ E. $\frac{3}{2}\int_0^4 f(x)\,dx$

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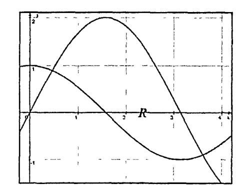
131. **(Calc)** Let $R$ be the region between the graphs of $y = 2\sin x$ and $y = \cos x$ as shown in the figure to the right. The region $R$ is the base of a solid with cross sections perpendicular to the $x$-axis as rectangles that are twice as high as wide. Find the volume of the solid.




A. 4.472 B. 7.854 C. 8.944

D. 15.708 E. 31.416

## R. Derivative of Accumulation Function ($2^{nd}$ FTC)

**What you are finding:** You are looking at problems in the form of $\frac{d}{dx}\int_a^x f(t)\,dt$. This is asking for the rate of change with respect to $x$ of the accumulation function starting at some constant (which is irrelevant) and ending at that variable $x$. It is important to understand that this expression is a function of $x$, not the variable $t$. In fact, the variable $t$ in this expression could be any variable (except $x$).

**How to find it:** You are using the $2^{nd}$ Fundamental Theorem of Calculus that says: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Occasionally you may have to use the chain rule that says $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$.

99. Let $f$ be the function given by $f(x)=\int_x^{\pi/2} t\sin 2t\,dt$ for $-\pi \le x \le \pi$. In what interval(s) is $f(x)$ decreasing?

A. $(0,\pi)$ B. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ C. $\left(-\pi,\frac{-\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$

D. $(-\pi,0)$ E. $(-\pi,\pi)$

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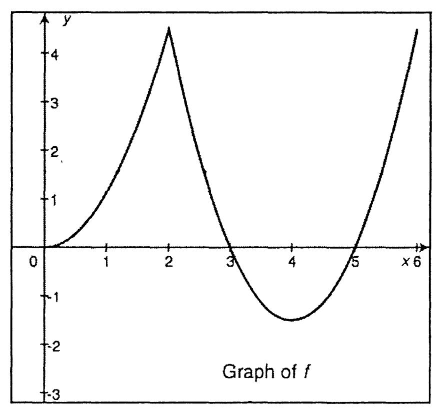
100. The graph of the function $f$ shown to the right has a horizontal tangent at $x = 4$. Let $g$ be the continuous function defined by $g(x)=\int_0^t f(t)\,dt$. For what value(s) of $x$ does the graph of $g$ have a point of inflection?




A. 3 and 5 B. 0, 3, and 5 C. 2 only

D. 2 and 4 E. 4 only

101. $\dfrac{d}{dx}\displaystyle\int_{1}^{x^3} \tan\left(t^4 - 1\right)\, dt =$

A. $\sec^2\left(x^{12} - 1\right)$ B. $\tan\left(x^4 - 1\right)$ C. $\tan\left(x^{12} - 1\right)$
D. $3x^2 \tan\left(x^{12} - 1\right)$ E. $12x^{11} \tan\left(x^{12} - 1\right)$

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102. **(Calc)** Let $f$ be the function given by $f(x) = \displaystyle\int_{0}^{x} \cos\left(t^2 + t\right)\, dt$ for $-2 \le x \le 2$. Approximately, for what percentage of values of $x$ for $-2 \le x \le 2$ is $f(x)$ decreasing?

A. 30% B. 26% C. 44% D. 50% E. 59%

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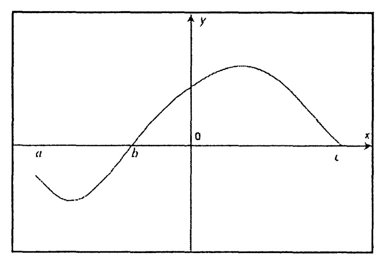
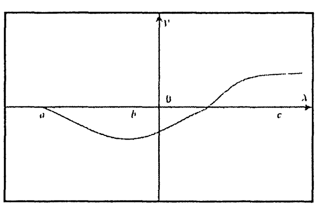
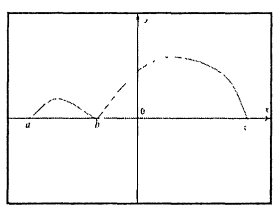
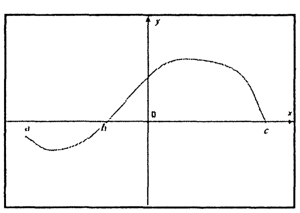
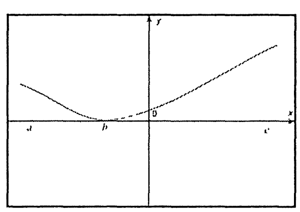
103. Let $f(x) = \displaystyle\int_{a}^{x} g(t)\, dt$ where $g$ has the graph shown to the right. Which of the following could be the graph of $f$?

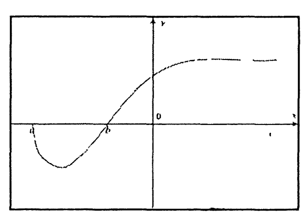


A. 

B. 

C. 

D. 

E. 

## H. Intermediate Value and Mean Value Theorem (MVT)

**What it says: (IVT)** If you have a continuous function on $[a,b]$ and $f(b) \neq f(a)$, the function must take on every value between $f(a)$ and $f(b)$ at some point between $x = a$ and $x = b$. For instance, if you are on a road traveling at 40 mph and a minute later you are traveling at 50 mph, at some time within that minute, you must have been traveling at 41 mph, 42 mph, and every possible value between 40 mph and 50 mph.

**What it says: (MVT)** If $f(x)$ is continuous on $[a,b]$ and differentiable on $(a, b)$, there must be some value of $c$ between $a$ and $b$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. In words, this says that there must be some value between $a$ and $b$ such that that the tangent line to the function at that value is parallel to the secant line between $a$ and $b$.

42. The function $f$ is continuous and non-linear for $-3 \le x \le 7$ and $f(-3) = 5$ and $f(7) = -5$. If there is no value $c$, where $-3 < c < 7$, for which $f'(c) = -1$, which of the following statements must be true?

A. For some $k$, where $-3 < k < 7$, $f'(k) < -1$.
B. For some $k$, where $-3 < k < 7$, $f'(k) > -1$.
C. For some $k$, where $-3 < k < 7$, $f'(k) = 0$.
D. For $-3 < k < 7, f'(k)$ exists.
E. For some $k$, where $-3 < k < 7$, $f'(k)$ does not exist.

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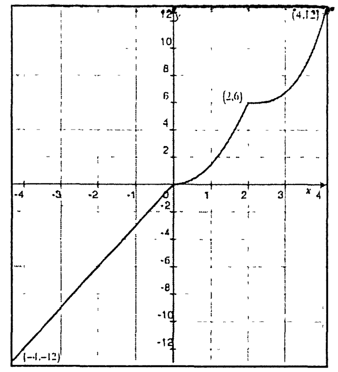
43. A new robotic dog called the IPup went on sale at (9 AM) and sold out within 8 hours. The number of customers in line to purchase the IPup at time $t$ *is* modeled by a differentiable function $A$ where $0 \le t \le 8$. Values of $A(t)$ are shown in the table below. For $0 \le t \le 8$, what is the fewest number of times at which $A'(t) = 0$?

| $t$ (hours) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $A(t)$ people | 150 | 185 | 135 | 120 | 75 | 75 | 100 | 120 | 60 |

A. 0   B. 2   C. 3   D. 4   E. 5

44. A continuous function $f$ is defined on the closed interval $-4 \le x \le 4$. The graph of the function, shown in the figure to the right consists of a line and two curves. There is a value $a$, $-4 \le a < 4$, for which the Mean Value Theorem, applied to the interval $[a, 4]$ guarantees a value $c$, $a \le c < 4$ at which $f'(c) = 3$. What are possible values of $a$?




I. -4 II. 0 III. 2

A I only
B. II only
C. III only
D. II and III only
E. I, II, and III

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45. Let $f$ be a twice-differentiable function such that $f(a) = b$ and $f(b) = a$ for two unknown constants $a$ and $b$, $a < b$. Let $g(x) = f(f(x))$. The Mean Value Theorem applied to $g'$ on $[a, b]$ guarantees a value $k$ such that $a < k < b$ such that

A. $g'(k) = 0$
B. $g''(k) = 0$
C. $g'(k) = 1$
D. $g''(k) = 1$
E. $g'(k) = b - a$

---

46. **(Calc)** There are value(s) of $c$ that satisfy the Mean-Value Theorem for $f(x) = 2\cos x - 4\cos 2x$ on $[0, \pi]$. Find the sum of these values.

A. 1.455
B. 4.493
C. 4.596
D. 1.687
E. -1.273

Name ______________ Date ______________ # ____

## Q. Fundamental Theorem of Calculus (FTC) / Accumulation Function

**What you are finding:** Students should certainly know that the FTC says that $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F(x)$ is an antiderivative of $f(x)$. This leads to the fact that $F(a) + \int_a^b f(x)\,dx$. Students should be prepared to use substitution methods to integrate and be able to change the limits of integration using this substitution. Students should also know that $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.

The accumulation function looks like this: $F(x) = \int_0^x f(t)\,dt$. It represents the accumulated area under the curve *f* starting at zero (or some value) and going out to the value of *x*. The variable *t* is a dummy variable. It is important to believe that this is a function of $x$.

92. $\int_0^{\pi} e^{\cos x} \sin x\,dx =$

A. $\frac{1}{e}$ B. $e - \frac{1}{e}$ C. $e - 1$ D. $1 - \frac{1}{e}$ E. $e$

---

93. $\int_1^3 \sqrt{20 - 4x}\,dx$ is equivalent to

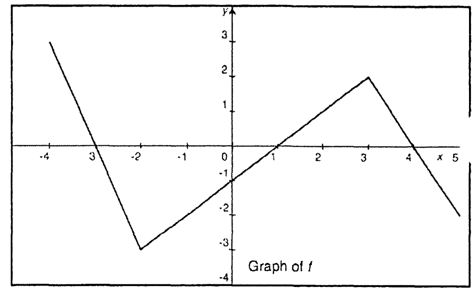
A. $\frac{-1}{4}\int_{-1/5}^{1/5} \sqrt{u}\,du$ B. $\frac{-1}{4}\int_1^3 \sqrt{u}\,du$ C. $\frac{1}{4}\int_8^{16} \sqrt{u}\,du$ D. $4\int_8^{16} \sqrt{u}\,du$ E. $\int_8^{16} \sqrt{u}\,du$

---

94. $\int_1^x \frac{\ln t}{t}\,dt =$

A. $\frac{x^2 - 1}{2}$ B. $\frac{x^2}{2}$ C. $\frac{(\ln x)^2}{2}$ D. $\frac{(\ln x)^2 - 1}{2}$ E. $\ln x$

95. The graph of the piecewise linear function $f$ is shown in the figure to right. If $g(x) = \int_1^x f(t)\,dt$, which of the following is the greatest?




A. $g(-4)$ B. $g(-3)$ C. $g(1)$
D. $g(4)$ E. $g(5)$

---

96. **(Calc)** Let $F(x)$ be an antiderivative of $\sqrt{x^3 + x + 1}$. If $F(1) = -2.125$, then $F(4) =$

A. -15.879 B. -11.629 C. 7.274 D. 15.879 E. 11.629

---

97. If $f$ is a continuous function and $F'(x) = f(x)$ for all real numbers $x$, then $\int_{-2}^{2} f(1-3x)\,dx =$

A. $3F(-2) - 3F(2)$ B. $\frac{1}{3}F(-2) - \frac{1}{3}3F(2)$ C. $\frac{1}{3}F(2) - \frac{1}{3}3F(-2)$

D. $3F(-5) - 3F(7)$ E. $\frac{1}{3}F(7) - \frac{1}{3}F(-5)$

---

98. If $f$ is continuous for all real numbers $x$ and $\int_1^4 f(x)\,dx = 10$, then $\int_3^6 \left[f(x-2) + 2x\right] dx =$

A. 37 B. 39 C. 35 D. 57 E. 25

## O. Computation of Riemann Sums

**What you are finding:** Riemann sums are approximations for definite integrals, which we know represent areas under curves. There are numerous real-life models for areas under curves so this is an important concept. Typically these types of problems show up when we are given data points as opposed to algebraic functions.

**How to find them:** Given data points:

| $x$ | $x_0$ | $x_1$ | $x_2$ | ... | $x_{n-2}$ | $x_{n-1}$ | $x_n$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $f(x_0)$ | $f(x_1)$ | $f(x_2)$ | ... | $f(x_{n-2})$ | $f(x_{n-1})$ | $f(x_n)$ |

Assuming equally spaced $x$-values: $x_{i+1} - x_i = b$

Left Riemann Sums: $S = b[f(x_0) + f(x_1) + f(x_2) + ... + f(x_{n-1})]$

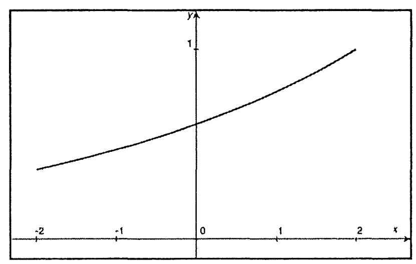
Right Riemann Sums: $S = b[f(x_1) + f(x_2) + f(x_3) + ... + f(x_n)]$

Trapezoids: $S = \frac{b}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$

If bases are not the same (typical in AP questions), you have to compute the area of each trapezoid: $\frac{1}{2}(x_{i+1} - x_i)[f(x_i) + f(x_{i+1})]$

Midpoints: This is commonly misunderstood. For example, you cannot draw a rectangle halfway between $x = 2$ and $x = 3$ because you may not know $f(2.5)$. You can't make up data. So in the table above the first midpoint rectangle would be drawn halfway between $x_0$ and $x_2$ which is $x_1$. So assuming that the $x$-values are equally spaced, the midpoint sum is $S = 2b[f(x_1) + f(x_3) + f(x_5) + ...]$.

83. The graph of the function $f$ is shown to the right for $-2 \le x \le 2$. Four calculations are made:




LS – Left Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

RS – Right Riemann sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

TS – Trapezoidal sum approximation of $\int_{-2}^{2} f(x)\,dx$ with 4 subintervals of equal length

DI – $\int_{-2}^{2} f(x)\,dx$

Arrange the results of the calculations from highest to lowest.

A. DI – RS – TS - LS
B. RS – TS – DI - LS
C. TS – DI – LS - RS
D. LS – DI – TS - RS
E. RS – DI – TS - LS

Dixie Ross

# RELATED RATES PROJECT

Due:________________________

For this project, you will write an original related rates problem. The problem must be illustrated on a small poster, diorama, or power point. Power points must be printed out before class for another student to work in class. The problem could have a seasonal theme (Halloween, Thanksgiving, Christmas, Hanukkah, Winter Solstice etc. – does not need to be religious in nature). The problem should utilize a mathematical relationship (Pythagorean Theorem, cone, similar triangles, sphere, circle, trig ratios, etc.). Make a 9 point rubric on regular sized paper with the problem correctly worked out. Your problems will be graded on the following criteria:

- 10 points-problem is creative, neat, original and entertaining
- 10 points-problem utilizes correct mathematical relationships
- 10 points-solution is correctly and clearly presented
- 10 points-illustrations (NOT computer generated!)/presentation
- 5 points-working correctly another student's project
- 5 points-difficulty level of the problem

HAVE FUN WITH THIS!! By the way, a well-done problem <u>may</u> appear on the Exam or Final Exam, and its author will receive Bonus Points!!

Name ______________________ Date ______________________ # ___

## C. Taking Derivatives with Basic Functions

**What you are finding:** The derivative of a function is a formula for the slope of the tangent line to the graph of that function. Students are required to know how to take derivative of basic functions, trig functions, logarithmic and exponentials, and inverse trig functions. They also need to be able to take derivative of inverse functions. Finally, rules such as power, product, quotient, chain rule, and taking derivatives implicitly must be a process that students have down perfectly.

**How to find it:**

Power Rule: $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Product Rule: $\frac{d}{dx}\left[f(x)\cdot g(x)\right] = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left[g(x)\right]^2}$

Chain Rule: $\frac{d}{dx}\left[f(g(x))\right] = f'\left[g(x)\right]\cdot g'(x)$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad \frac{d}{dx}(\cos x) = -\sin x$ $\quad \frac{d}{dx}(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}(\csc x) = -\csc x \cot x$ $\quad \frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad \frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad \frac{d}{dx}\left(e^x\right) = e^x$ $\quad \frac{d}{dx}\left(a^x\right) = a^x \ln a$ $\quad \frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

11. If $f(x) = (2x+1)\left(x^2-3\right)^4$, then $f'(x) =$

A. $2\left(x^2-3\right)^3\left(x^2+4x-1\right)$ B. $4(2x+1)\left(x^2-3\right)^3$ C. $8x(2x+1)\left(x^2-3\right)^3$

D. $2\left(x^2-3\right)^3\left(3x^2+x-3\right)$ E. $2\left(x^2-3\right)^3\left(9x^2+4x-3\right)$

---

12. If $f(x) = \ln\left(\frac{e}{x^n}\right)$, and $n$ is a constant, then $f'(x) =$

A. $\frac{-n}{x}$ B. $\frac{x^n}{e}$ C. $\frac{-1}{x^n}$ D. $\frac{e}{x^n}$ E. 0

---

13. If $f(x) = \sqrt[3]{\cos^2(3x)}$, then $f'(x) =$

A. $\frac{-2}{3\sqrt[3]{\sin 3x}}$ B. $\frac{-2}{\sqrt[3]{\sin 3x}}$ C. $\frac{-2\sin 3x}{\sqrt[3]{\cos 3x}}$ D. $\frac{2}{\sqrt[3]{\cos 3x}}$ E. $\frac{-2\sin 3x}{3\sqrt[3]{\cos 3x}}$

14. The table below gives value of the differentiable function $f$ and $g$ at $x=-1$. If $h(x)=\dfrac{f(x)-g(x)}{2f(x)}$, then $h'(-1)=$

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| $-1$ | $-2$ | $4$ | $e$ | $-3$ |

A. $\dfrac{-e-3}{4}$ B. $\dfrac{e+3}{2e}$ C. $\dfrac{e-6}{8}$ D. $\dfrac{2e-3}{4}$ E. $\dfrac{-4e-3}{4}$

---

15. The functions $f$ and $g$ are differentiable and $f(g(x))=x^2$ for all $x$. If $f(4)=8,\ g(4)=8,\ f'(8)=-2$, what is the value of $g'(4)$?

A. $\dfrac{-1}{8}$ B. $\dfrac{-1}{2}$ C. $-2$ D. $-4$ E. Insufficient data

---

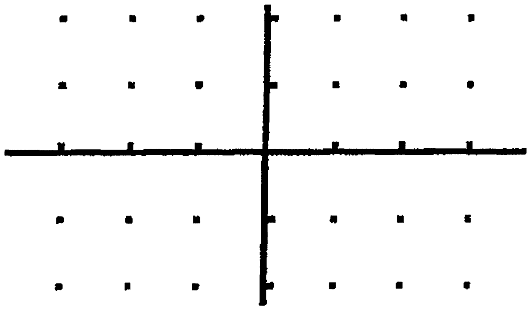
16. $f(x)=e^{2\ln\tan(x^2)}$, then $f'(x)=$

A. $e^{2\ln\tan(x^2)}$ B. $\dfrac{e^{2\ln\tan(x^2)}}{x^2}$ C. $\dfrac{e^{2\ln\tan(x^2)}}{2\tan x^2}$ D. $\dfrac{2\sin(x^2)}{\cos^3(x^2)}$ E. $\dfrac{4x\sin(x^2)}{\cos^3(x^2)}$

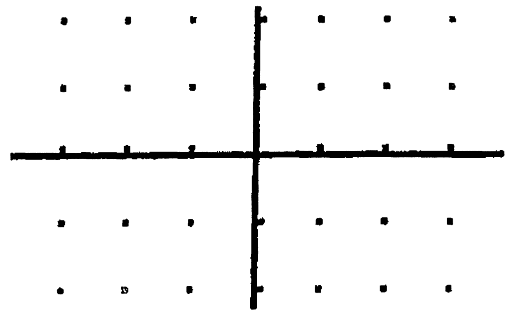
# SLOPE FIELDS

Draw a slope field for each of the following differential equations.

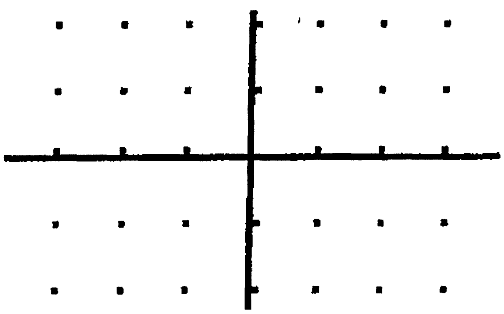
1. $\frac{dy}{dx} = x+1$



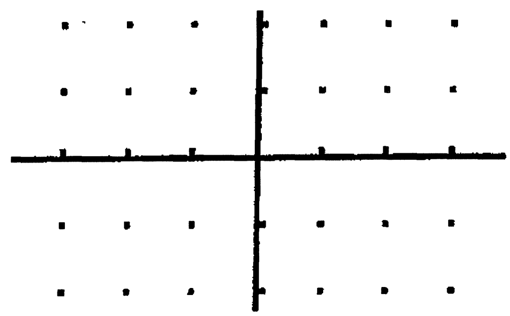
2. $\frac{dy}{dx} = 2y$



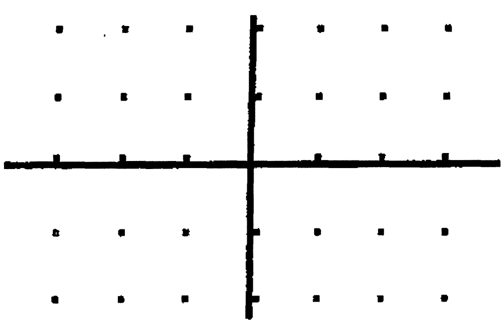
3. $\frac{dy}{dx} = x+y$



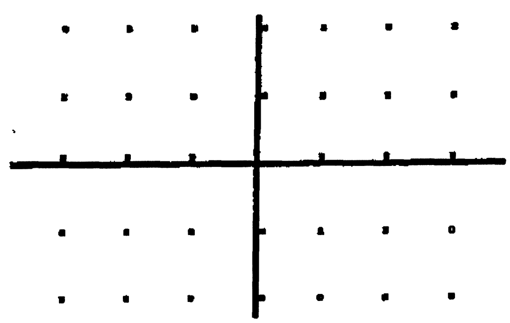
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y-1$

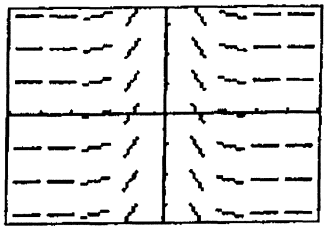


6. $\frac{dy}{dx} = -\frac{y}{x}$

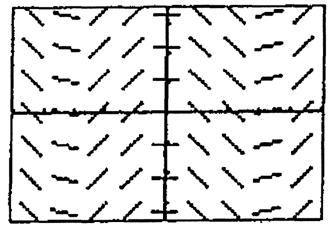


Match each slope field with the equation that the slope field could represent.

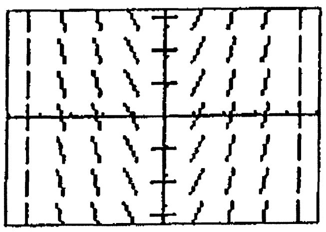
(A)



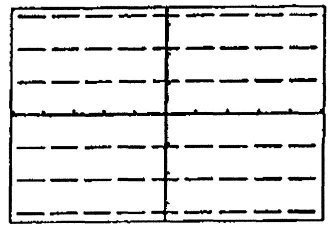
(B)



(C)



(D)



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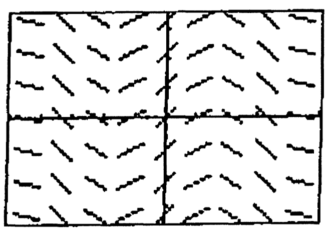
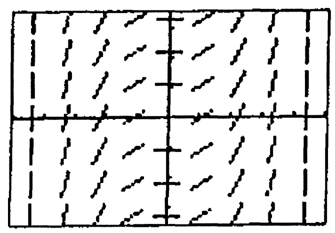
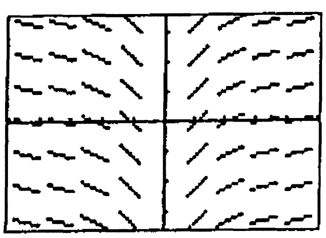


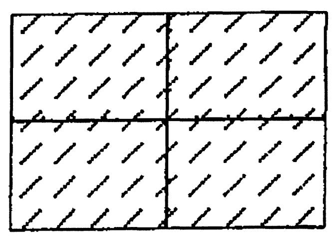
(F)



(G)



(H)



7. $y = 1$

8. $y = x$

9. $y = x^2$

10. $y = \frac{1}{6}x^3$

11. $y = \frac{1}{x^2}$

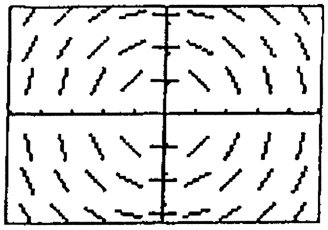
12. $y = \sin x$

13. $y = \cos x$

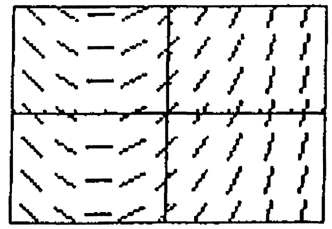
14. $y = \ln|x|$

Match the slope fields with their differential equations.

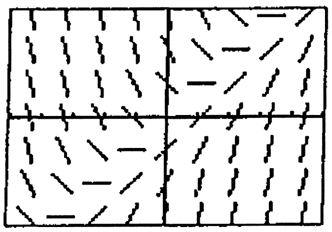
(A)



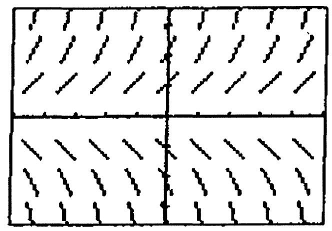
(B)



(C)



(D)



15. $\dfrac{dy}{dx}=\dfrac{1}{2}x+1$

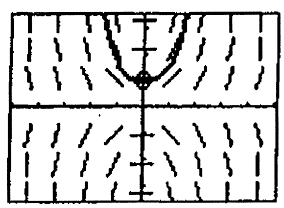
16. $\dfrac{dy}{dx}=y$

17. $\dfrac{dy}{dx}=x-y$

18. $\dfrac{dy}{dx}=-\dfrac{x}{y}$

---

19. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx}=xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
   (a) Sketch the solution curve through the point (0, 2).
   (b) Sketch the solution curve through the point $(0, -1)$.



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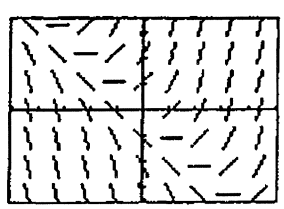
20. The calculator drawn slope field for the differential equation $\dfrac{dy}{dx}=x+y$ is shown in the figure below.
   (a) Sketch the solution curve through the point (0, 1).
   (b) Sketch the solution curve through the point $(-3, 0)$.



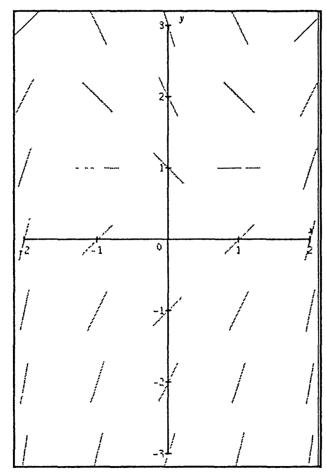
## W. Differential Equations

**What you are finding:** A differential equation (DEQ) is in the form of $\frac{dy}{dx} = (\text{Algebraic Expression})$. The goal of solving a DEQ is to work backwards from the derivative to the function; that is to write an equation in the form of $y = f(x) + C$ (since the technique involves integration, the general solution will have a constant of integration). If the value of the function at some value of $x$ is known (an initial condition), the value of $C$ can be found. This is called a specific solution.

**How to find it:** In Calculus AB, the only types of DEQ's studied are called separable. Separable DEQ's are those than that can be in the form of $f(y)\,dy = g(x)\,dx$. Once in that form, both sides can be integrated with the constant of integration on only one side, usually the right.

Word problems involving change with respect to time are usually models of DEQ's. A favorite type is a problem using the words: The rate of change of $y$ is proportional to some expression. The equation that describes this statement is: $\frac{dy}{dt} = k \cdot (\text{expression})$. The rate of change is usually with respect to time.
Usually there is a problem that requires you to create a slope field. Simply calculate the slopes using the given derivative formula and plot them on the given graph. Usually the slopes will be integer values.

132. The slope field for the equation in the figure to the right could be



A. $\frac{dy}{dx} = x + y^2$ B. $\frac{dy}{dx} = x - y^2$ C. $\frac{dy}{dx} = xy$

D. $\frac{dy}{dx} = x + y$ E. $\frac{dy}{dx} = x^2 - y$

---

133. If $\frac{dy}{dx} = 3x^2y^2$ and if $y = 1$ when $x = 2$, then when $x = -1$, $y =$

A. $\frac{1}{10}$ B. $\frac{1}{8}$ C. $\frac{1}{19}$ D. $\frac{-1}{8}$ E. -1