HACT Workshop
Derivatives, Slope Fields *and more*

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AP Calculus Resource Site

http://online.math.uh.edu/apcalculus/

Over 40 Online Quizzes and AP Practices Exams

AP Calc

http://www.estudy.uh.edu/

Some Free Stuff

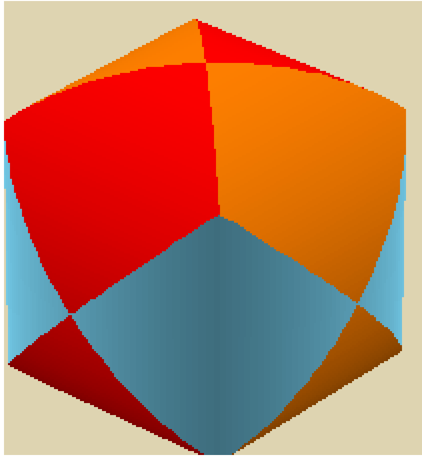
- Online resources for Calculus and Finite Math: http://www.zweigmedia.com/RealWorld/utilsindex.html
- Sage: http://sagemath.org
- Geogebra: http://www.geogebra.org/cms/
- Winplot: http://math.exeter.edu/rparris/winplot.html
- GraphFunc Online: http://graph.seriesmathstudy.com/
- SpaceTime: http://www.spacetime.us/
- Graph: http://www.padowan.dk

- Use online
- Can save worksheets
- http://sagemath.org
- http://wiki.sagemath.org/interact/calculus
- http://www.sagemath.org/library.html
- http://www.sagemath.org/doc/reference/
- http://wiki.sagemath.org/quickref

GeoGebra

- Allows you to create dynamic worksheets for the web (ex: http://www.math.uh.edu/~bekki/sectan.html)
- Easy to use
- Free for both PC and Mac
- 5.0 has built in CAS and 3-D
- http://www.geogebra.org/en/wiki/index.php/Release_Notes_GeoGebra_5.0
- http://www.geogebra.org/webstart/5.0/geogebra-50.jnlp
- http://www.geogebra.org/webstart/5.0/GeoGebra-Windows-Portable-3-9-5-0.zip

Winplot



- http://math.exeter.edu/rparris/winplot.html
- Easy to use with 2-D and 3-D
- PC only

GraphFunc Online

- http://graph.seriesmathstudy.com/
- Online only
- 2-D and 3-D views
- Need latest JRE

SpaceTime

- http://www.spacetime.us/
- Free for PC but $$ for Mac and iPhone/iPad
- Great graphics
- Powerful
- Tutorial during first download

Graph

- http://www.padowan.dk
- 2-D only
- Download free for PC only (won’t run on Mac)

Break!

www.math.uh.edu/~bekki/HACT_101511.pdf

Derivatives

Some problems:

Derivative defined as the limit of the difference quotient.

Evaluate the limit:

or if ↙ is $f'(a)$, what is $f(x)$ and a?

$$\lim_{h \to 0} \frac{\frac{1}{(2+h)} - \frac{1}{2}}{h}$$

$f(x) = \frac{1}{x}$

$f'(2) = ?$

$$\lim_{h \to 0} \frac{5\cos\left(\frac{1}{4}\pi + h\right) - 5\cos\left(\frac{1}{4}\pi\right)}{h}$$

$= f'(\pi/4)$ for

$f(x) = 5\cos(x)$

$$\lim_{h \to 0} \frac{\sqrt{3+h} - \sqrt{3}}{h}$$

$f(x) = \sqrt{x}$

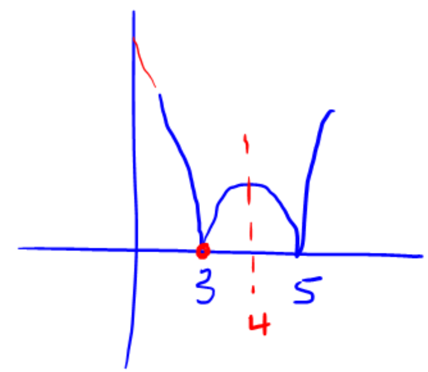
$f'(3)$

The position of a particle moving along a line is given by $s(t) = t^3 - 12t^2 + 45t + 4$

for $0 \le t \le 4$. For what value(s) of t is the speed of the particle smallest?

$v(t) = 3t^2 - 24t + 45$

$|v(t)| = |3(t-3)(t-5)|$



The function g is differentiable, and the tangent line to the graph of $y = g(x)$ at $x = -2$ is $y = 4x - 2$. Let $f(x) = 2g(x) + 3x + 2$. Give $f'(-2)$.

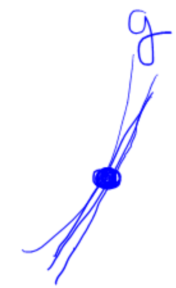
$g'(-2)$

$$f'(x) = 2g'(x) + 3$$

$$f'(-2) = 2g'(-2) + 3 = 2(4) + 3 = 11$$

The function *g* is differentiable, and the tangent line to the graph of $y = g(x)$ at $x = -1$ is $\boxed{y = 2x - 1}$. Let $f(x) = 2[g(x)]^2 + 2x + 4$. Give $f\,'(-1)$.

$g'(-1)$



$$f'(x) = 4[g(x)] \cdot g'(x) + 2$$

$$f'(-1) = 4(g(-1)) \cdot g'(-1) + 2$$

$$= 4(-3)(2) + 2$$

Suppose that f is a differentiable function, $f(-2) = 3$, and $f\,'(-2) = 2$. Find $g\,'(-2)$ given that

$$g(x) = 3\,(f(x) - 3)^2 - \frac{4}{f(x)}$$

$$g'(x) = 6\,(f(x) - 3)(f'(x)) + \frac{4f'(x)}{[f(x)]^2}$$

$$\frac{d}{dx}\left[[f(9x)]^2\right] = 2f(9x) \cdot f'(9x) \cdot 9$$

The functions f and g are differentiable and

$h(x) = f(g(x))$.

Use the information below to find $h'(4)$:

$f(4) = -3$, $f'(4) = 2$

$f(2) = -3$, $f'(2) = 3$

$g(4) = 2$, $g'(4) = -2$

$g(-3) = 3$, $g'(-3) = -2$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

The functions f and g are differentiable and

$h(x) = f(x)/g(x)$.

Use the information below to find $h'(3)$:

$f(3) = -1$, $f'(3) = 6$

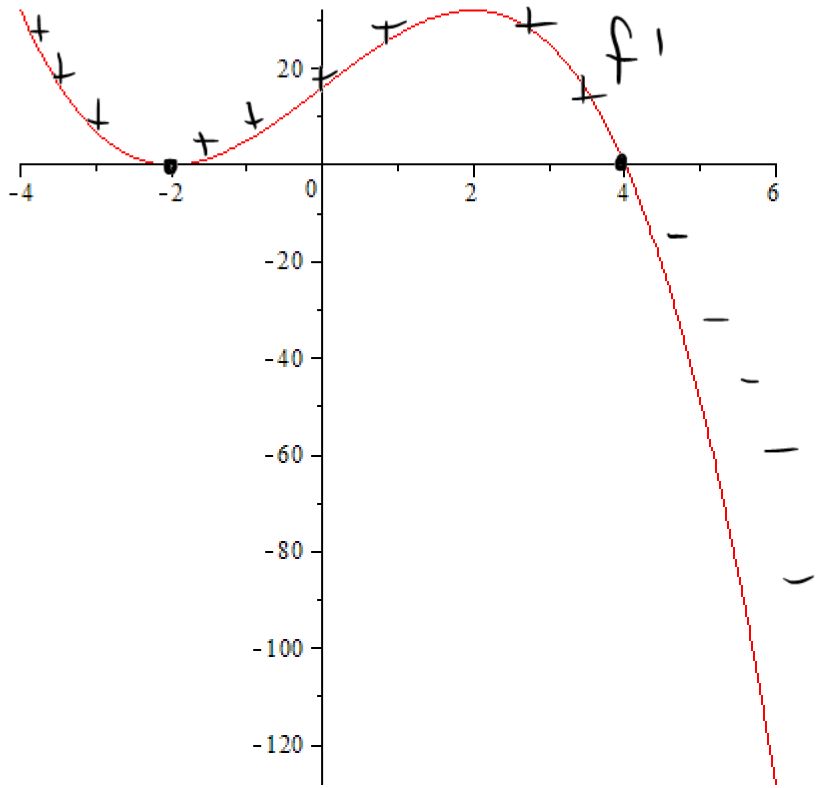
$f(6) = -4$, $f'(6) = 0$

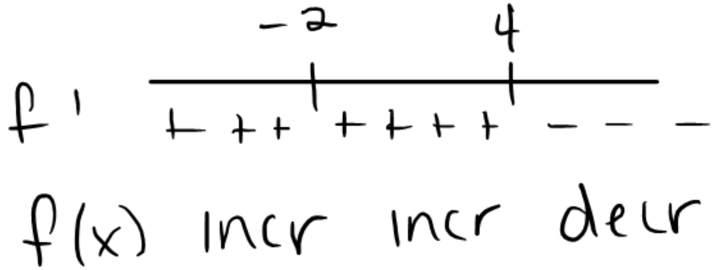
$g(3) = 6$, $g'(3) = -6$

$g(-4) = 4$, $g'(-4) = -6$

QR.

The graph of *f* ' (the derivative of *f*) is shown below. At what value of *x* does the graph of *f* (*x*) change from increasing to decreasing?





The graph of f is given below. If f is twice differentiable, then which of the following is true?

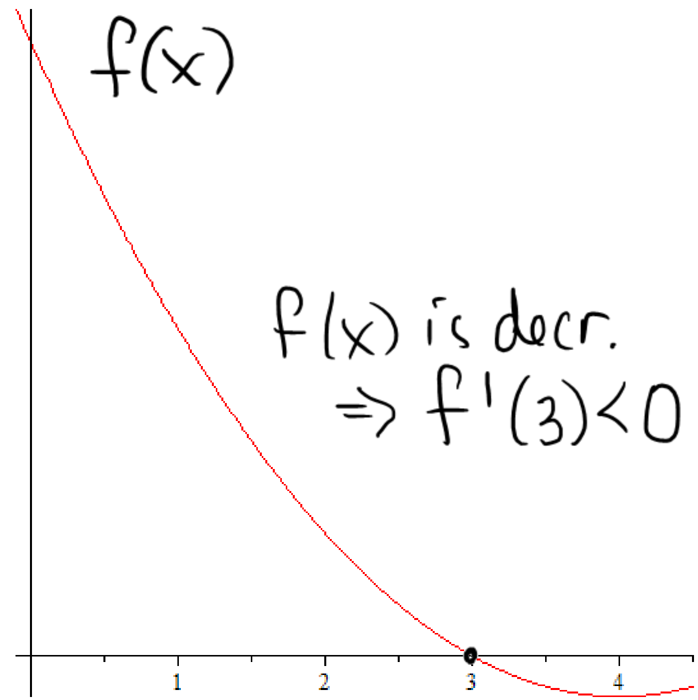
a) $f(3) < f''(3) < f'(3)$

☆b) $f'(3) < f(3) < f''(3)$

c) $f''(3) < f'(3) < f(3)$

d) $f''(3) < f(3) < f'(3)$

e) $f(3) < f'(3) < f''(3)$



f(x) is decr. $\Rightarrow f'(3) < 0$

$f'(3) \leq f(3) \leq f''(3)$

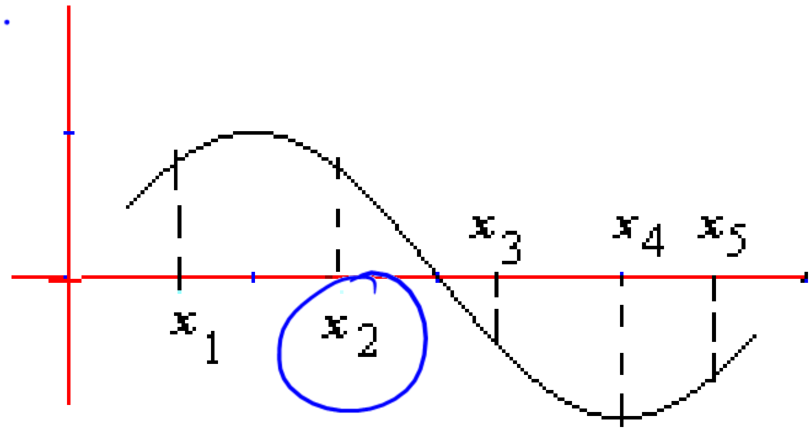
$f(3) = 0$

f(x) is concave up $f''(3) > 0$

The graph of $y = f(x)$ is given below. For which of the five domain values shown is $f''(x) < 0$ and $f'(x) < 0$?

Concave down

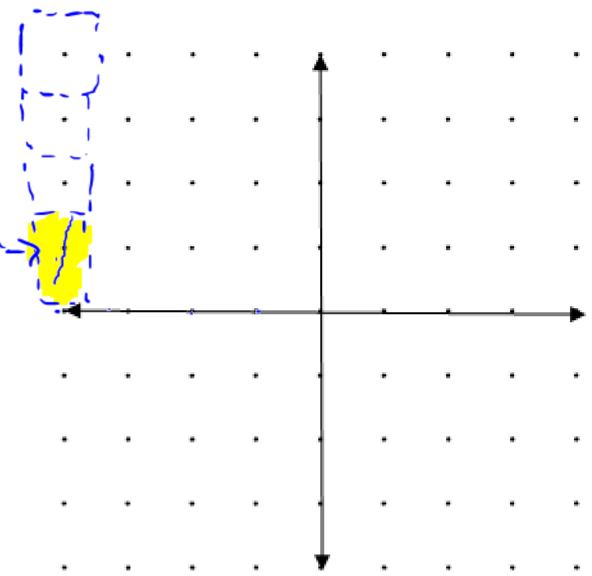
decr.



Slope Fields

(-4, 1)

Intro Activity

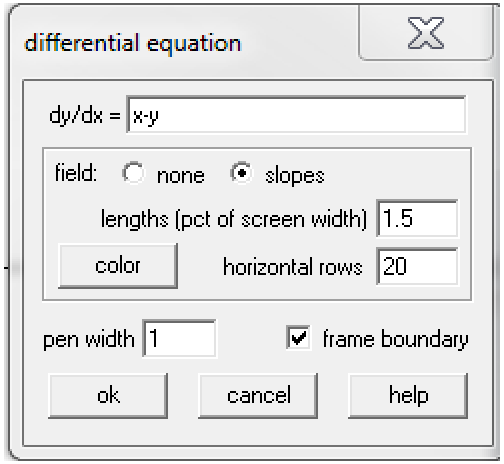


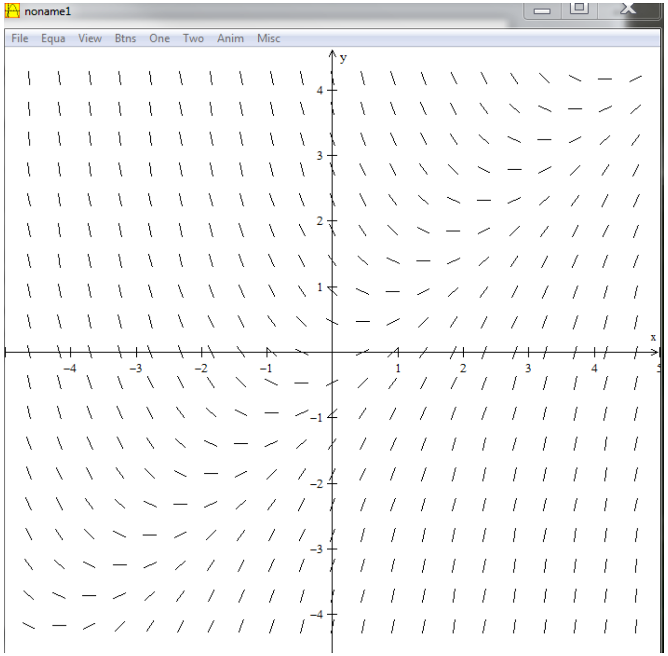
Give all students a small square cut from an overhead transparency with a dot on it. The dots will each represent a coordinate point. Then give the class a problem such as $\boxed{\frac{dy}{dx}=x^2}$ or $\frac{dy}{dx}=x^3$ and talk about what this means (slope of tangent line at the point). Have them each draw small tangent lines and come place on the overhead. Expand to more difficult differential equations (some they cannot integrate easily).

Slope = 16

Creating Slope Fields with Winplot

Equation->Differential->dy/dx=

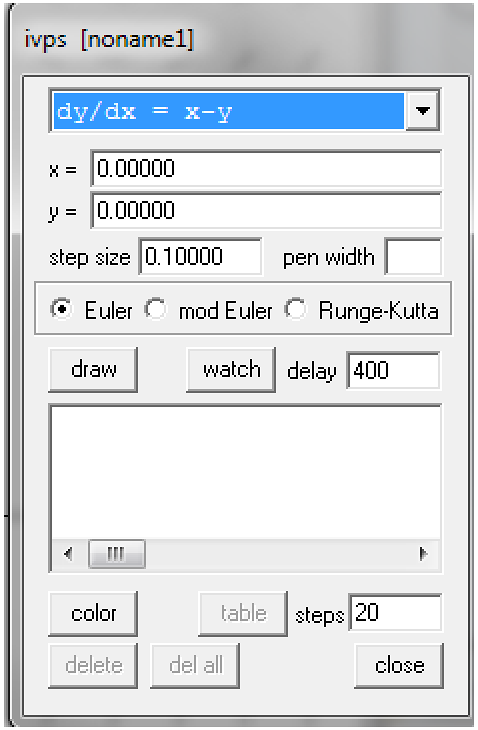


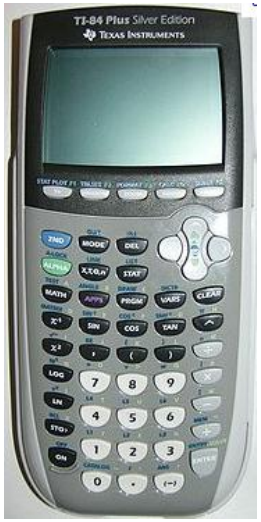


Now choose One->Initial Value Problem

Pick your x and y values then select draw:

also do stepsize -.1





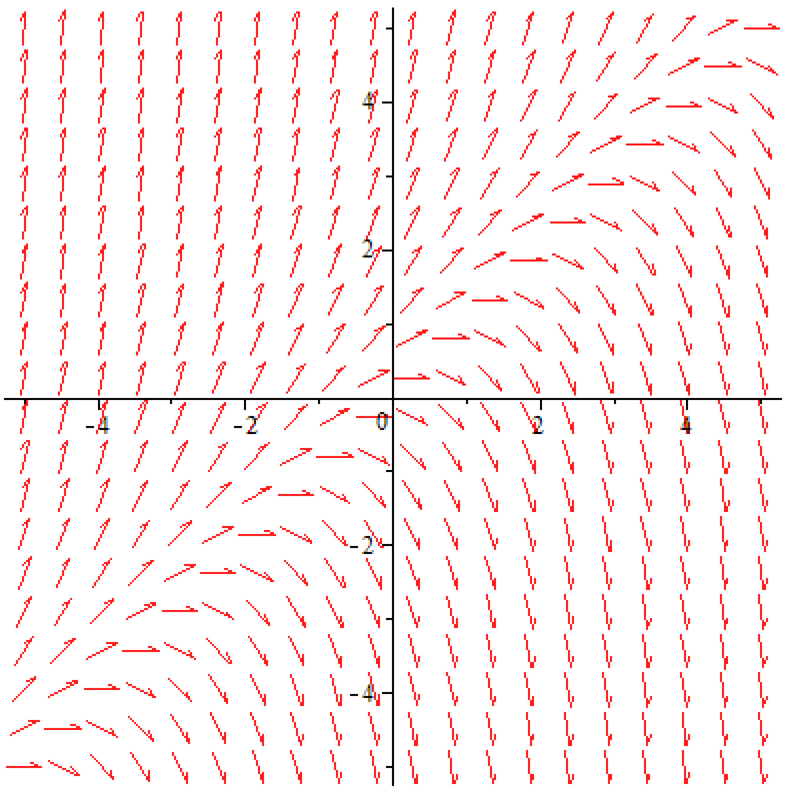
Ti Activities (some old ones):

http://education.ti.com/xchange/US/Math/Calculus/4369/Calculus_Acts_Act13.pdf

http://education.ti.com/xchange/US/Math/Calculus/4370/Calculus_Acts_Act14.pdf

Some Problems:

Which of the following differential equations has the following slope field?



a) $\frac{dy}{dx} = \ln(y)$

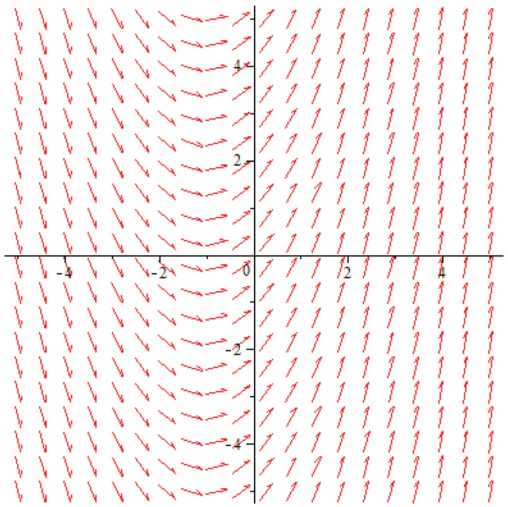
b) $\frac{dy}{dx} = x^2$

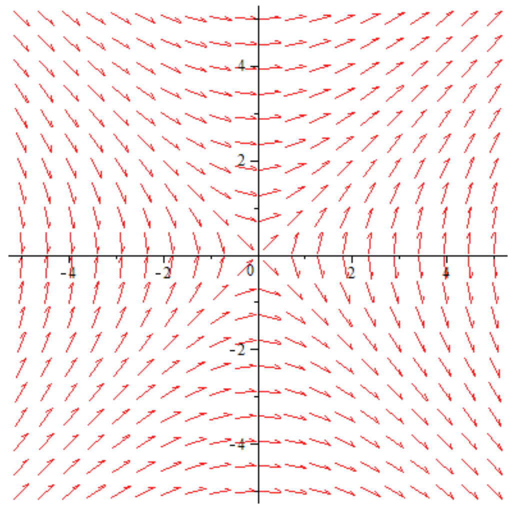
c) $\frac{dy}{dx} = y - x$

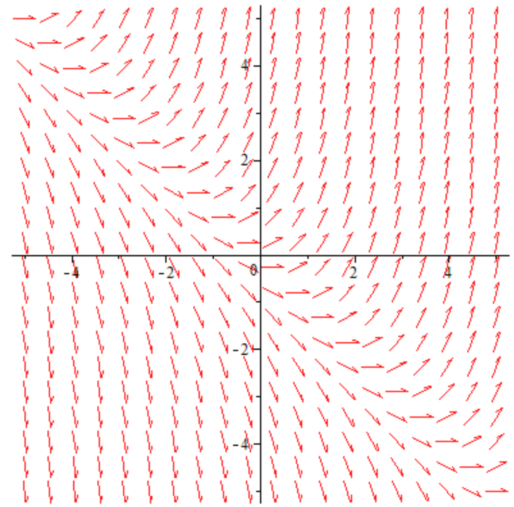
d) $\frac{dy}{dx} = \frac{x}{y}$

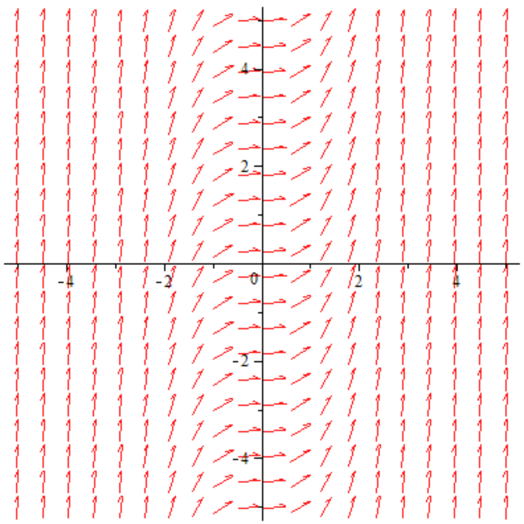
e) $\frac{dy}{dx} = x + 1$

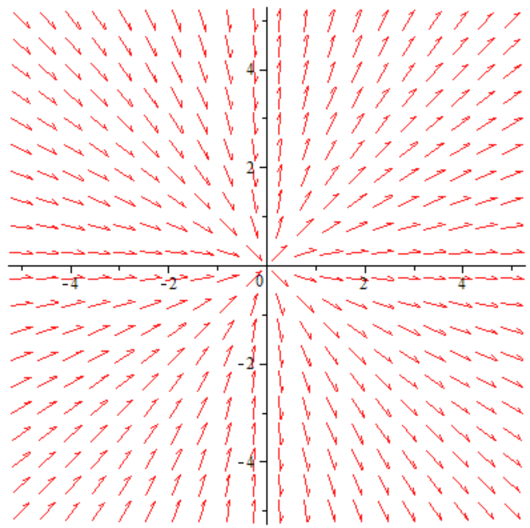
Which of the following is a slope field for the given differential equation? $\frac{dy}{dx} = \frac{x}{y}$



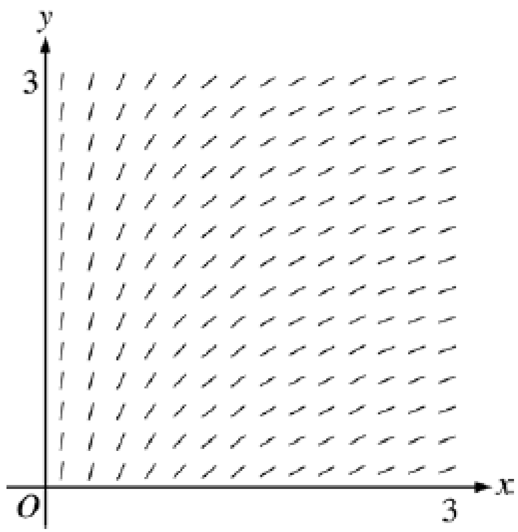








From Sample AP* exam



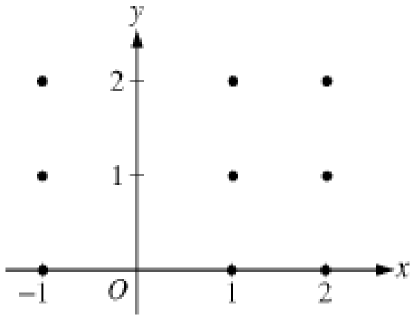
The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$
(B) $y = e^x$
(C) $y = e^{-x}$
(D) $y = \cos x$
(E) $y = \ln x$

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

A few more questions…

The function f is differentiable and $\int_0^x (6f(t) + 2t)\,dt = \cos(x)$ Determine the value of $f\,'(\pi/3)$.

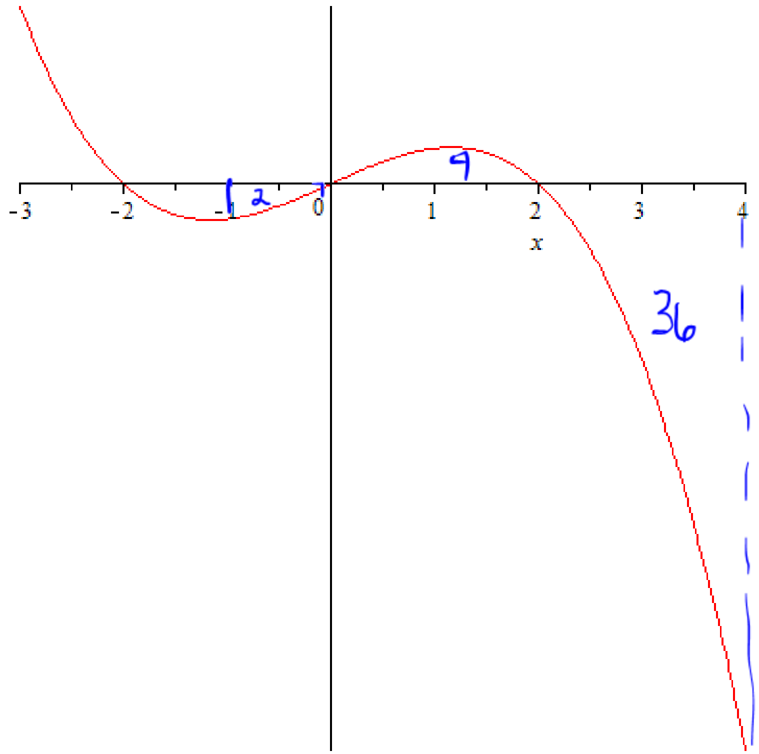
$$\frac{d}{dx}\int_0^x (6f(t) + 2t)\,dt = \frac{d}{dx}\cos x$$

$$6f(x) + 2x = -\sin x$$

$$f(x) = \frac{-\sin x - 2x}{6}$$

$$f'(x) =$$

$$f'(\pi/3)$$



$$\int_{-1}^{4} f(x)\,dx =$$

$$-2+4-36$$

The area of the region bounded between the graph of *f* and the *x*-axis from *x* = -1 to *x* = 0 is 2, from *x* = 0 to *x* = 2 is 4, and from *x* = 2 to *x* = 4 is 36. Evaluate: $\int_{-1}^{4} f(x)\,dx$