Using the TI Graphing Calculator on Piecewise Functions, Piecewise Derivatives, Area and Volume

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Can you define a piecewise function?

Evaluating piecewise functions:

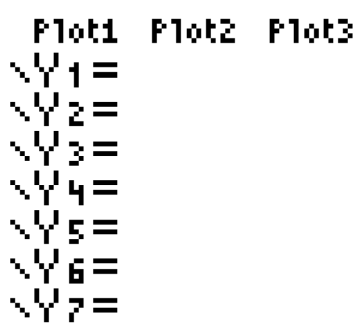
$$f(x) = \begin{cases} x^2 + 1 & x < 2 \\ 3 - x & 2 \le x \end{cases}$$

$$f(x) = \begin{cases} 2x - 3 & x < 2 \\ 5 & x = 2 \\ x + 1 & 2 < x \end{cases}$$

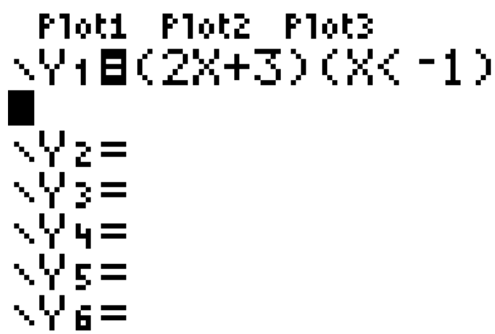
Graphing piecewise functions on TI-83/84:

$$f(x) = \begin{cases} 2x + 3 & x < -1 \\ x^2 & -1 \le x \textbf{ and } x \le 2 \\ 6 - x & 2 < x \end{cases}$$

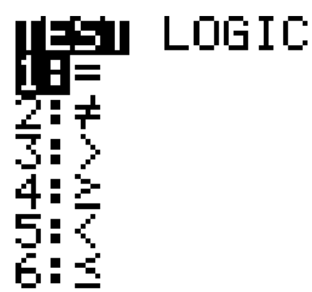
Choose Y=



Enter first function in ( ) with conditional next to it in ( )



Use 2nd Math for inequality symbols

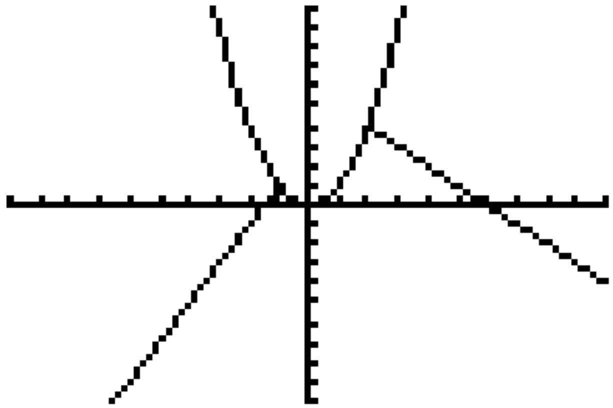


Lets graph what we have so far.

Suggestions on graphing other "pieces"?

What if:

```
 Plot1  Plot2  Plot3
\Y1=(2X+3)(X<-1)
\Y2=(X²)(-1≤X≤2)
\Y3=(6-X)(X>2)
\Y4=
\Y5=
```



We have a problem with the compound inequality (-1 ≤ x ≤ 2)
There are two ways to correct this – use one of the following:

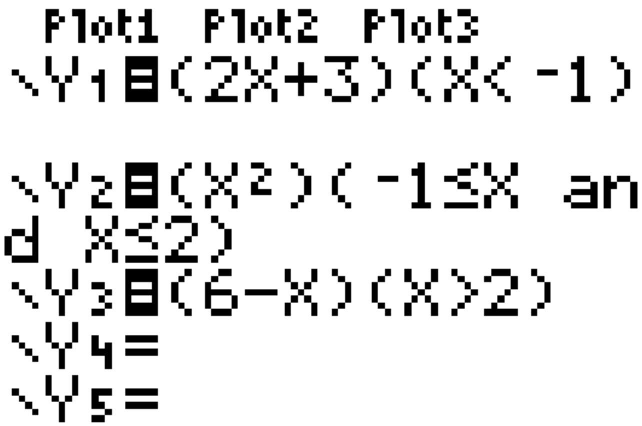
(-1 ≤ x)(x ≤ 2)

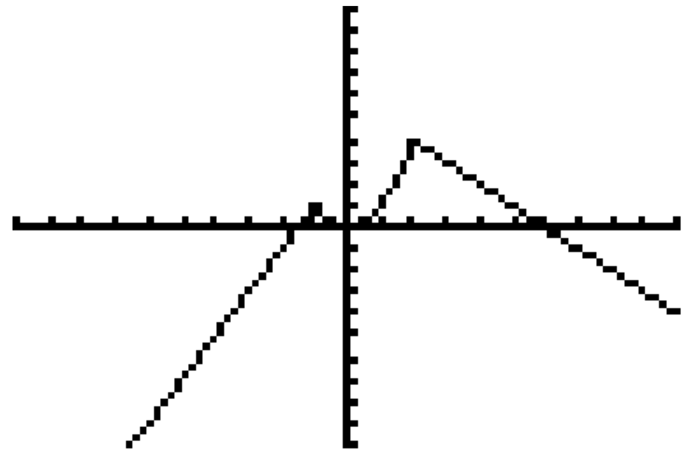
or

(-1 ≤ x and x ≤ 2)

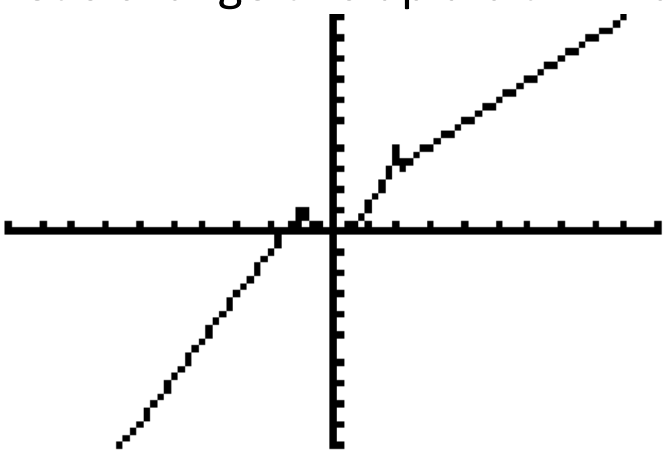
I like to use the second method. To get the “and” operator:

Now we have:





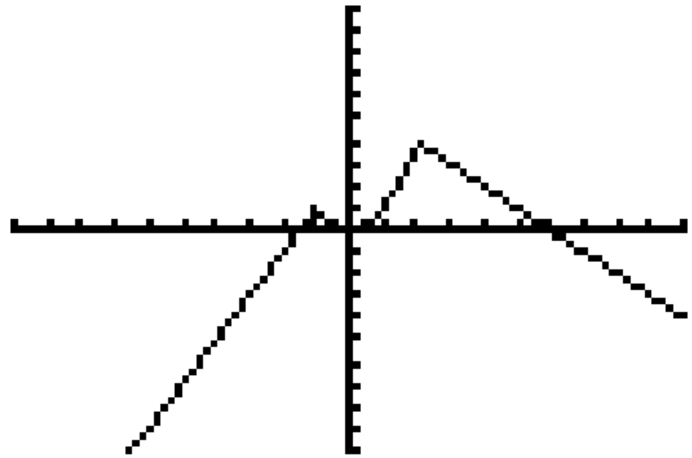
Let’s change this up a bit. What if the third “piece” was (x+1)?



Next, what if we want to evaluate different values for our function using the calculator?

We can make these 3 functions into one …..

```
Plot1 Plot2 Plot3
\Y1=(2X+3)(X<-1)
+(X^2)(-1≤X and X
≤2)+(6-X)(X>2)
\Y2=
\Y3=
\Y4=
\Y5=
```



Now we can evaluate any value with just one function:

```
Y1(-5)
                    -7
Y1(1)
                     1
Y1(8)
                    -2
```

How about a table:

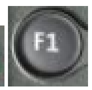
X	Y1	
-1.03	.94	
-1.02	.96	
-1.01	.98	
-1	1	
-.99	.9801	
-.98	.9604	
-.97	.9409	

X=-.97

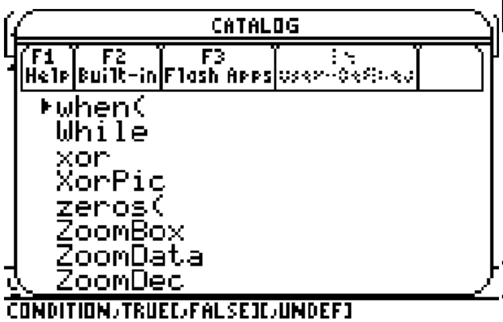
Note: your y-values may be rounded. If you arrow over to the y-value, it will show to more decimal places below.

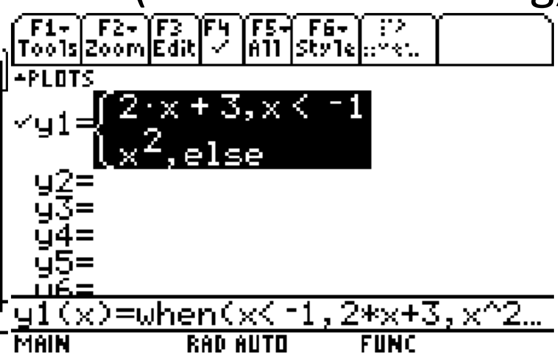
TI-89:

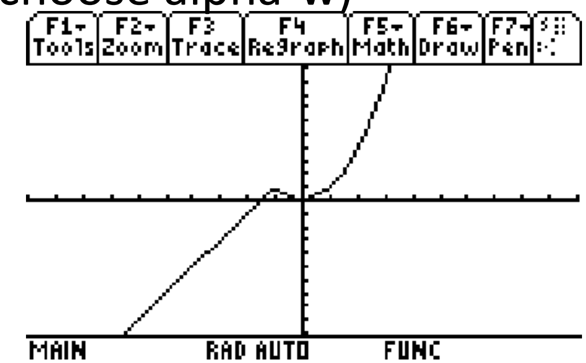
$$f(x)=\begin{cases} 2x+3 & x<-1 \\ x^2 & -1\le x \end{cases}$$

Press  and select y1=

Press  and then “when” (instead of scrolling, choose alpha-w)

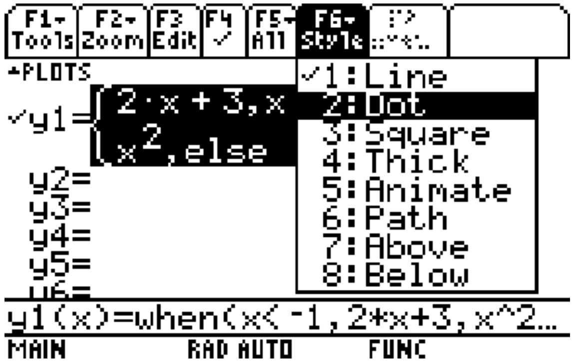






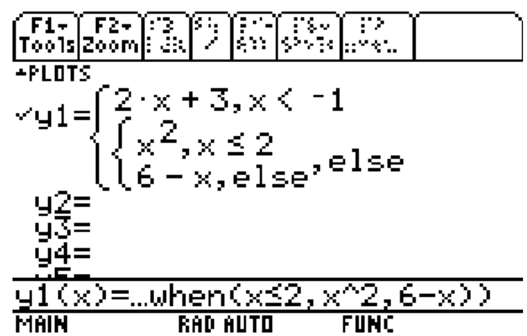
the < and > are located above ‘0’ and ‘.’

Note: Sometimes the TI calculators "connect" the graphs when they shouldn't. In this case, you want to be in "Dot" mode.



For the TI-89, if you have more than two pieces, you will need to have nested when statements:

$$f(x) = \begin{cases} 2x+3 & x < -1 \\ x^2 & -1 \le x \textbf{ and } x \le 2 \\ 6-x & 2 < x \end{cases}$$



Would be input as y1=when(x<-1,2*x+3,when(x<=2,x^2,6-x))

Let's try some more:

$$f(x) = \begin{cases} x - 4 & x < 1 \\ 2 - x^2 & 1 \leq x \end{cases}$$

$$f(x) = \begin{cases} 3 & x < -2 \\ x^3 & -2 \leq x \ \textbf{and} \ x < 3 \\ 2x + 1 & 3 \leq x \end{cases}$$

$$f(x) = |x|$$

Limits:

How can we use this with limits?

Given:

$$f(x)=\begin{cases} 2x-5 & x\neq 1 \\ 4 & x=1 \end{cases}$$

Find $\lim\limits_{x->1} f(x)$

Graph:

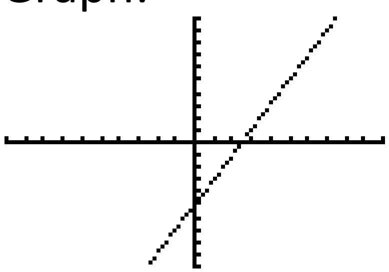
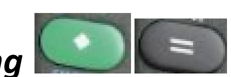


Table:

X	Y1	
.85	-3.3	
.9	-3.2	
.95	-3.1	
1	4	
1.05	-2.9	
1.1	-2.8	
1.15	-2.7	

X=.85

On the TI-89, enter y1=when(x$\neq$1,2x-5,4). The $\neq$ is obtained by pressing 

Continuity:

A function is *continuous* if

1.

2.

3.

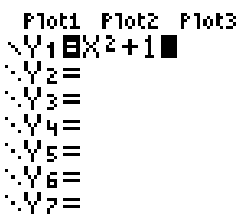
How can we apply what we talked about above to demonstrate this definition?

Area:

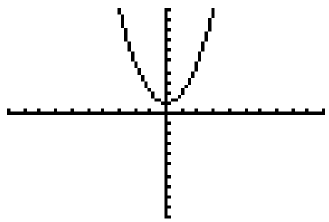
Graphing regions above the x-axis:

Ex: $f(x) = x^2 + 1$

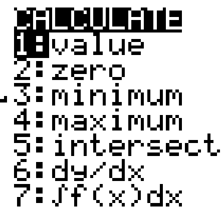
Enter function into y1



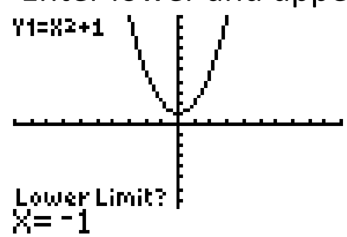
graph

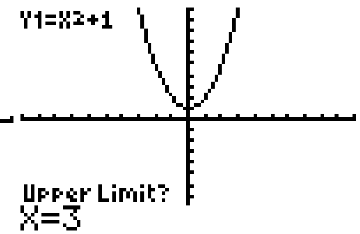


2nd Trace 7

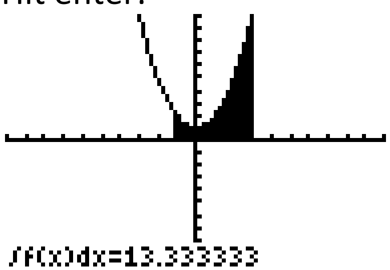


Enter lower and upper limit





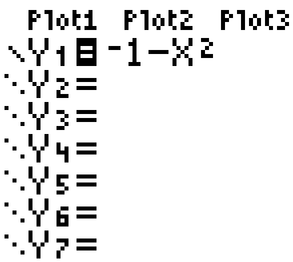
Hit enter:

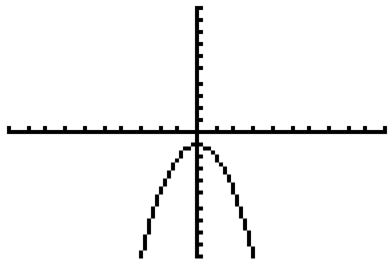


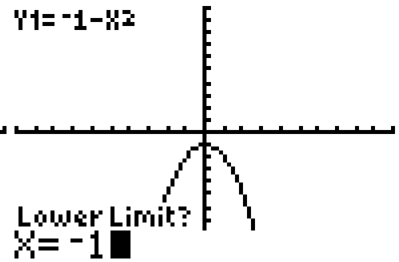
Not only does this shade the region, you have found the area.

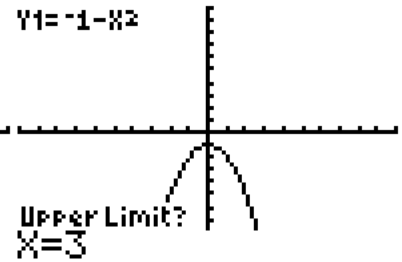
Now what if the graph is below the x-axis?

$$f(x) = -1 - x^2$$

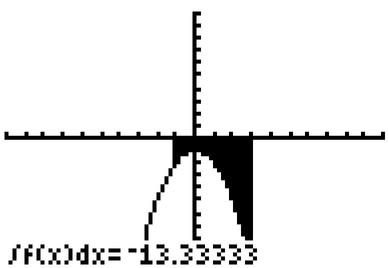






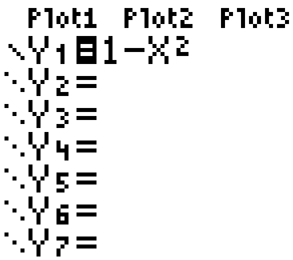


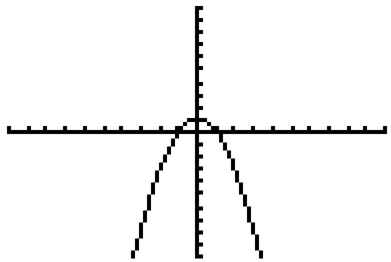
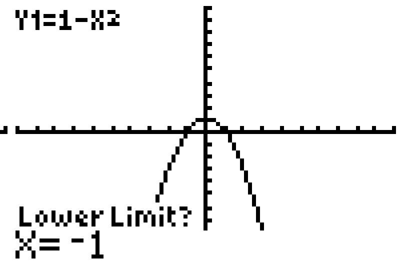
Here is the graph:

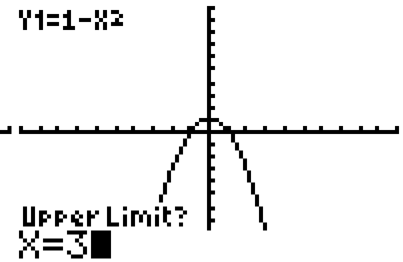


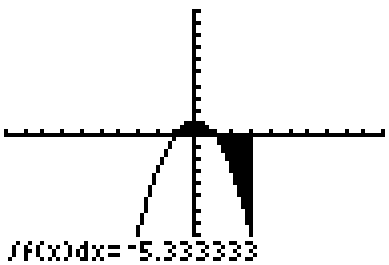
Now what about:

$f(x) = 1 - x^2$



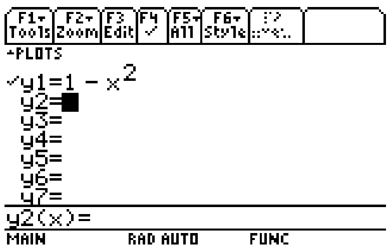





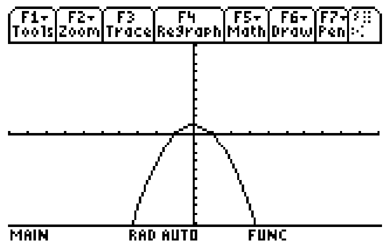


There are a couple of solutions to this problem. Let's discuss them.

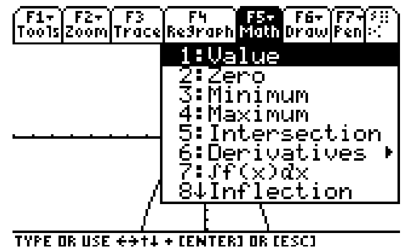
TI-89:



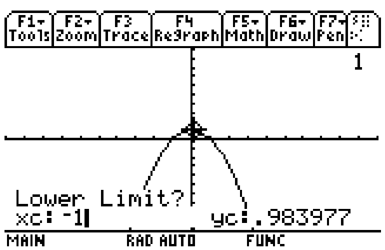
F1 Tools F2 Zoom F3 Edit F4 ✓ F5 All F6 Style
PLOTS
✓y1=1 − x^2
y2=
y3=
y4=
y5=
y6=
y7=
y2(x)=
MAIN RAD AUTO FUNC



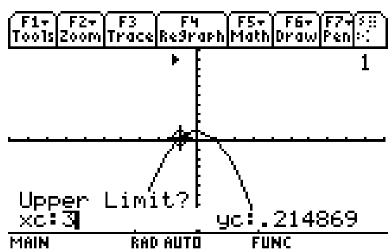
F1 Tools F2 Zoom F3 Trace F4 Regraph F5 Math F6 Draw F7 Pen
MAIN RAD AUTO FUNC



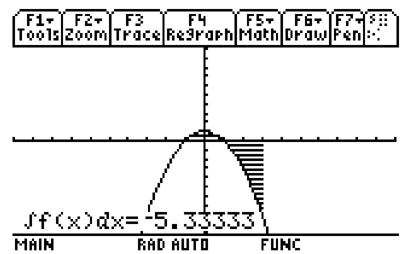
F1 Tools F2 Zoom F3 Trace F4 Regraph F5 Math F6 Draw F7 Pen
1:Value
2:Zero
3:Minimum
4:Maximum
5:Intersection
6:Derivatives
7:∫f(x)dx
8↓Inflection
TYPE OR USE ←→↑↓ + [ENTER] OR [ESC]



F1 Tools F2 Zoom F3 Trace F4 Regraph F5 Math F6 Draw F7 Pen
1
Lower Limit?
xc: -1
yc: .983977
MAIN RAD AUTO FUNC



F1 Tools F2 Zoom F3 Trace F4 Regraph F5 Math F6 Draw F7 Pen
1
Upper Limit?
xc: 3
yc: .214869
MAIN RAD AUTO FUNC



F1 Tools F2 Zoom F3 Trace F4 Regraph F5 Math F6 Draw F7 Pen
∫f(x)dx= -5.33333
MAIN RAD AUTO FUNC

Area between two curves:

Enter your functions into y1 and y2.
Lets use y1 = x^2 and y2 = x^3
Graph and verify which on is the lower function.

Use the Shade command (2nd – Draw – 7)

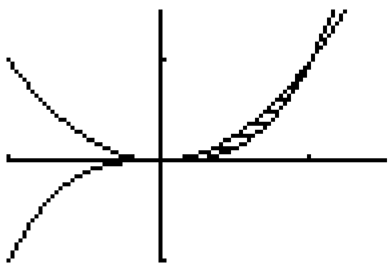
Parameters: shade(lower function, upper function, start, end, *pattern, partes*)

pattern=**1** vertical (default)
pattern=**2** horizontal
pattern=**3** negative—slope 45°
pattern=**4** positive—slope 45°

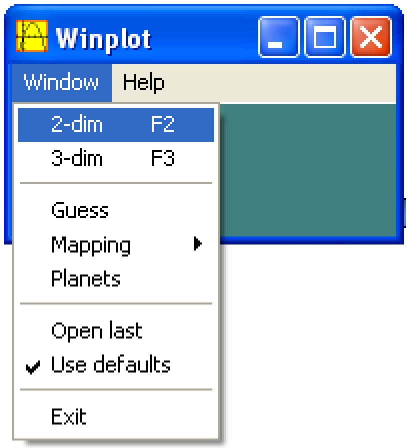
patres specifies one of eight shading resolutions.
patres=**1** shades every pixel (default)
patres=**2** shades every second pixel
patres=**3** shades every third pixel
patres=**4** shades every fourth pixel
patres=**5** shades every fifth pixel
patres=**6** shades every sixth pixel
patres=**7** shades every seventh pixel
patres=**8** shades every eighth pixel

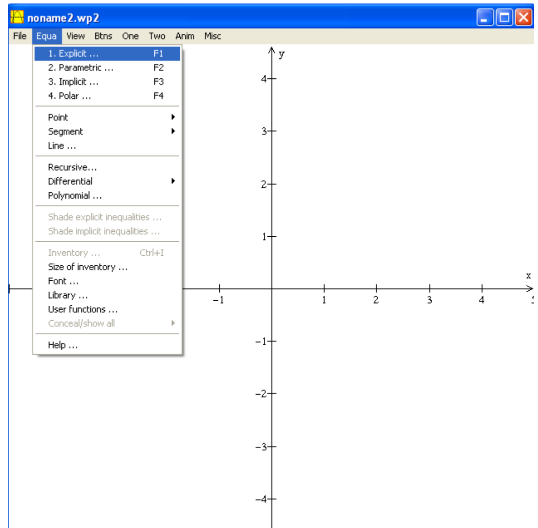
so, for our functions we will use:

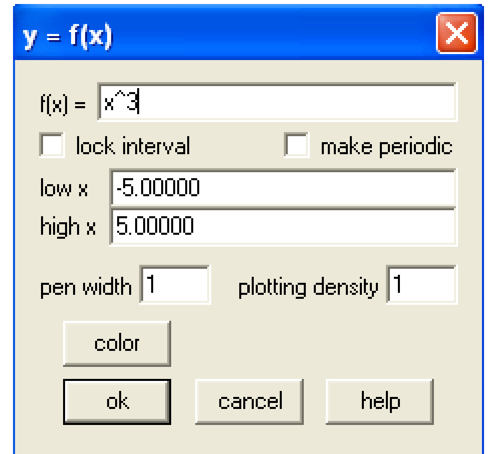
shade(Y2,Y1, 0, 1, 2, 3)

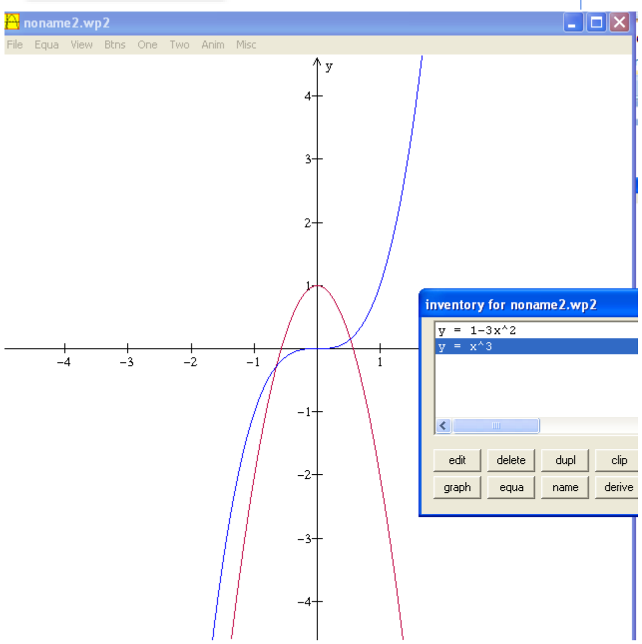


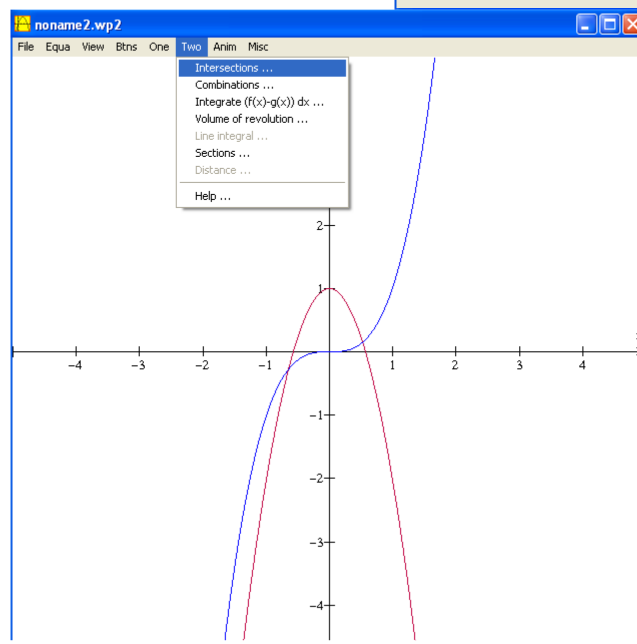
Using Winplot:

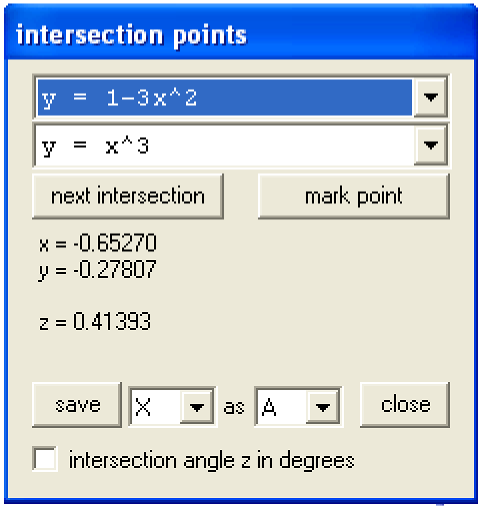


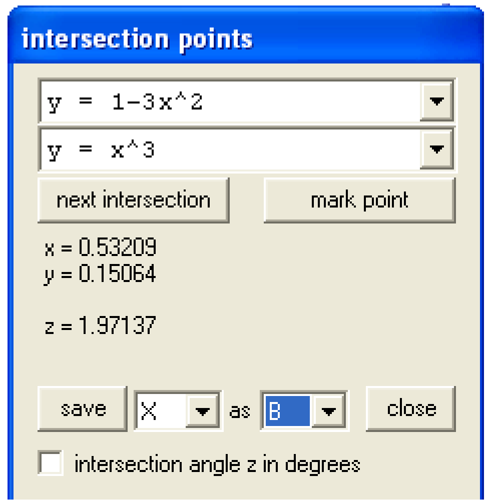


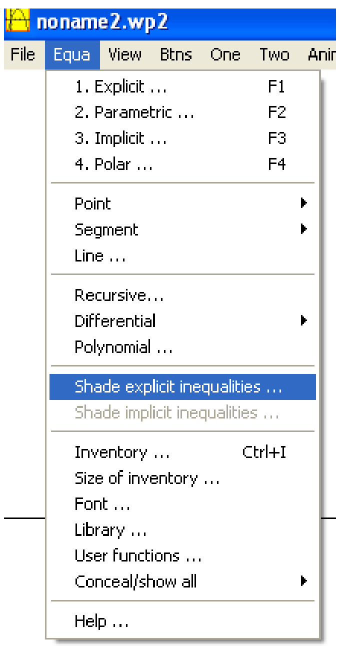


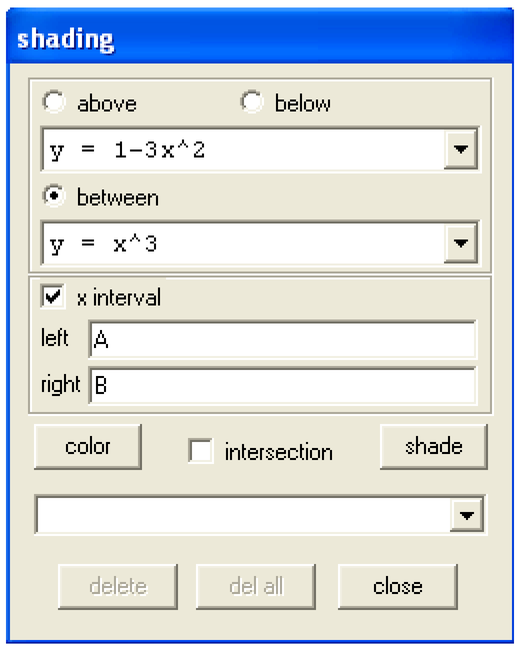


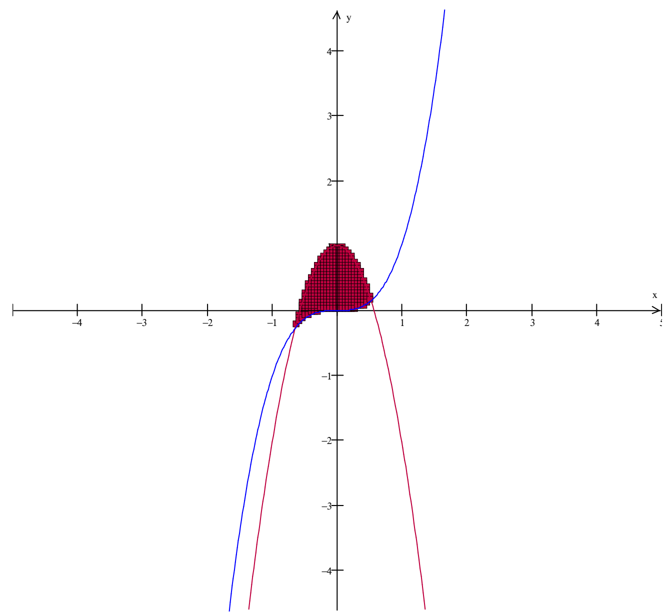


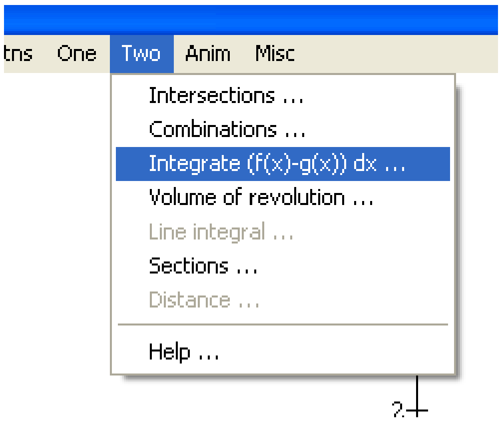

intersection points
y = 1-3x^2
y = x^3
next intersection
mark point
x = -0.65270
y = -0.27807
z = 0.41393
save
X
as
A
close
intersection angle z in degrees

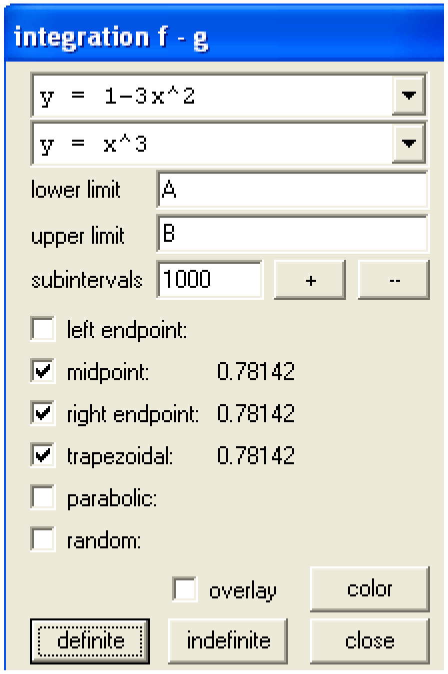

intersection points
y = 1-3x^2
y = x^3
next intersection
mark point
x = 0.53209
y = 0.15064
z = 1.97137
save
X
as
B
close
intersection angle z in degrees


noname2.wp2
File Equa View Btns One Two Anir
1. Explicit ... F1
2. Parametric ... F2
3. Implicit ... F3
4. Polar ... F4
Point
Segment
Line ...
Recursive...
Differential
Polynomial ...
Shade explicit inequalities ...
Shade implicit inequalities ...
Inventory ... Ctrl+I
Size of inventory ...
Font ...
Library ...
User functions ...
Conceal/show all
Help ...


shading
above
below
y = 1-3x^2
between
y = x^3
x interval
left A
right B
color
intersection
shade
delete
del all
close



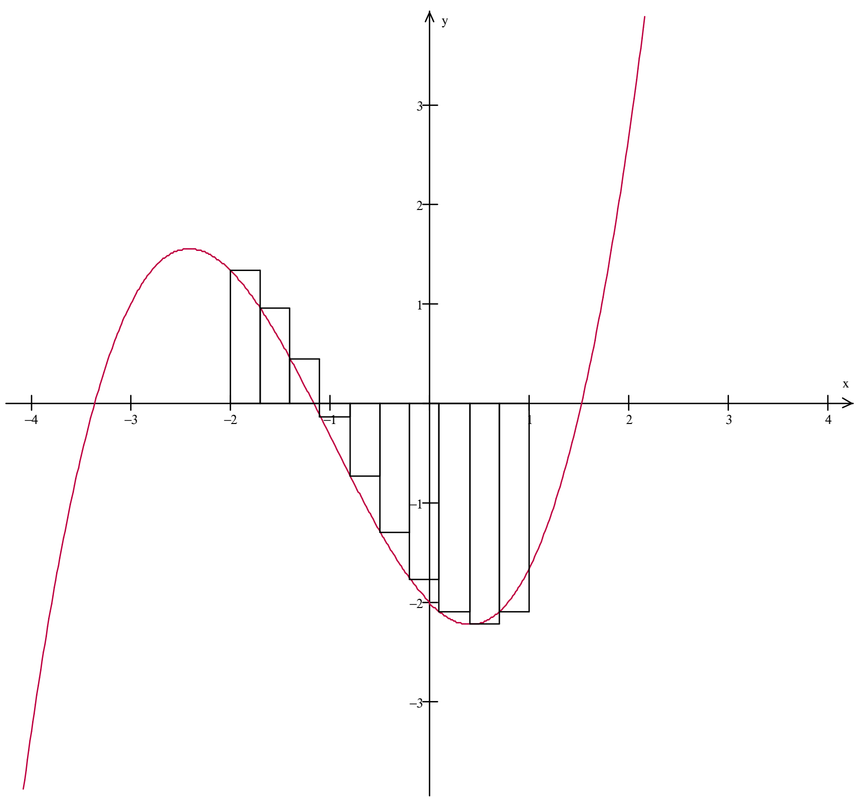

tns
One
Two
Anim
Misc
Intersections ...
Combinations ...
Integrate (f(x)-g(x)) dx ...
Volume of revolution ...
Line integral ...
Sections ...
Distance ...
Help ...


integration f - g
y = 1-3x^2
y = x^3
lower limit
A
upper limit
B
subintervals
1000
+
--
left endpoint:
midpoint: 0.78142
right endpoint: 0.78142
trapezoidal: 0.78142
parabolic:
random:
overlay
color
definite
indefinite
close

Riemann Sums:

Left-hand sums:

Ex: $f(x) = \frac{1}{3}x^3 + x^2 - x - 2$



In calculator:

y1 = f(x)

Δx = $\frac{b-a}{n}$ (width)

a = left endpoint

n = number of rectangles

$$\sum_{k=0}^{n-1} f(a + k \cdot \Delta x) \cdot \Delta x$$

Enter function into y1 =

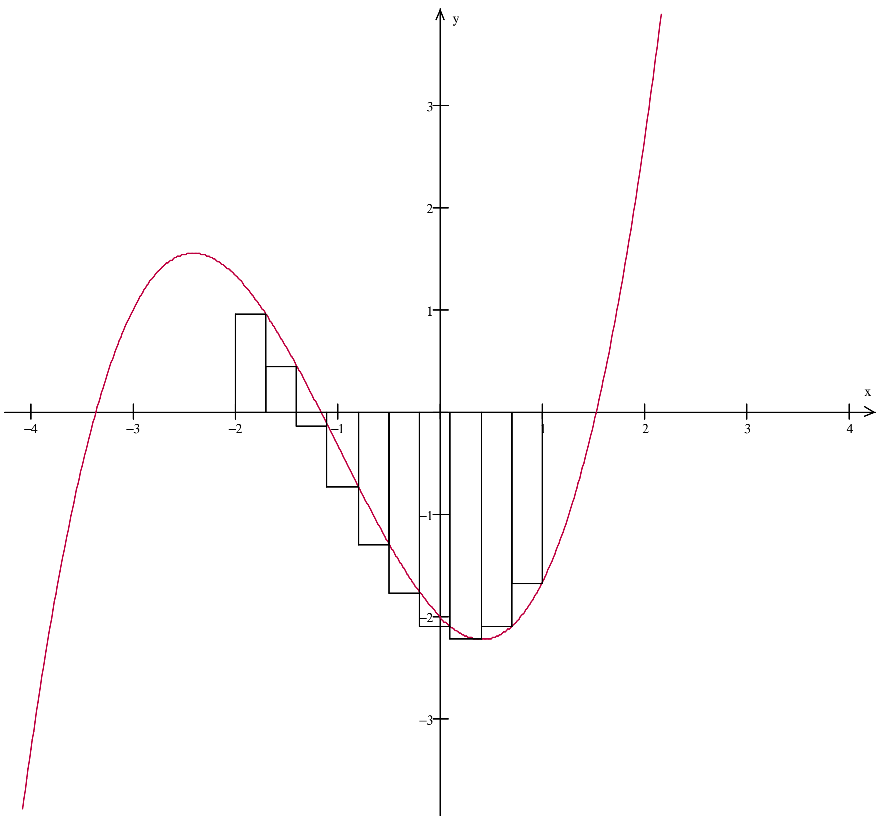
Use: sum(seq(y1(a+k*w)*w,k,0,n-1))

sum – List – Math - 5

seq – List – Ops – 5

Right-hand sums:

Ex: $f(x)=\frac{1}{3}x^3+x^2-x-2$



In calculator:

y1 = f(x)

Δx = $\frac{b-a}{n}$ (width)

a = left endpoint

n = number of rectangles

$$\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

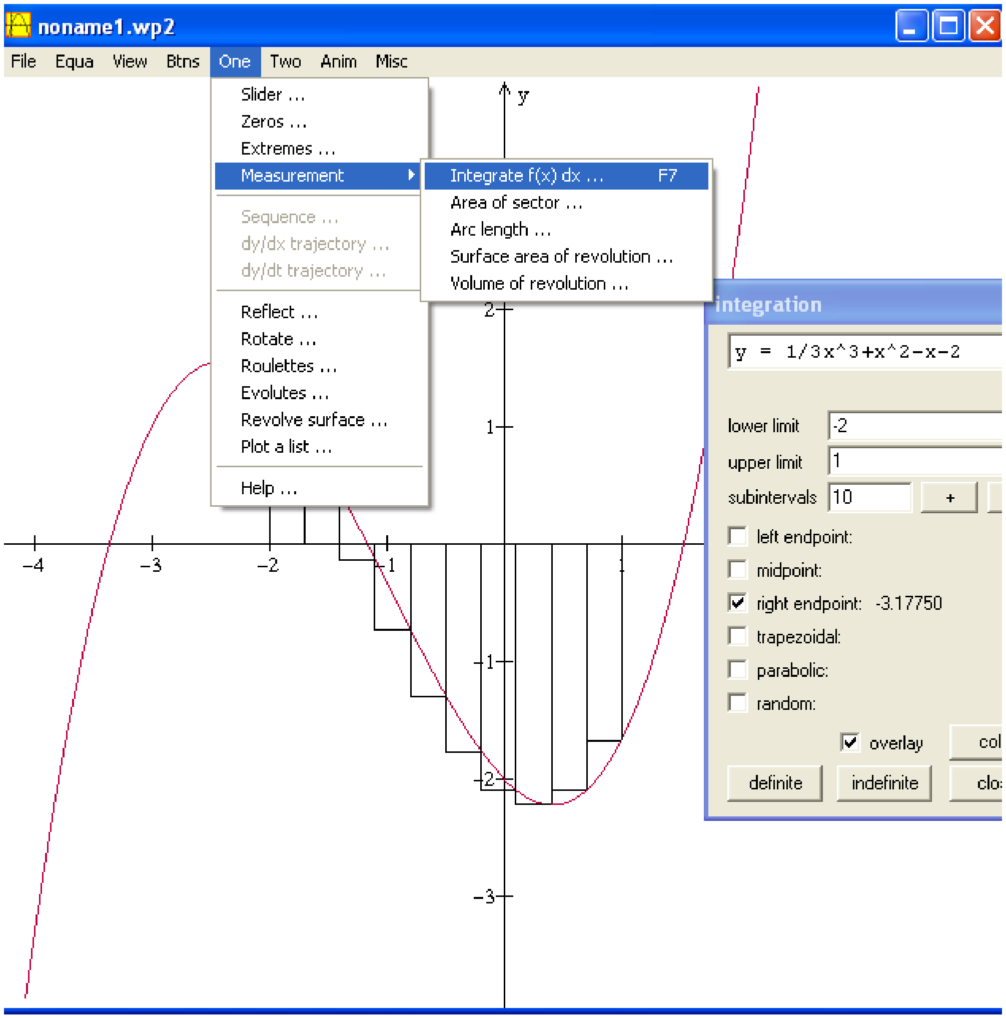
Enter function into y1 =

Use: sum(seq(y1(a+k*w)*w,k,1,n))

sum – List – Math - 5

seq – List – Ops – 5

Using Winplot:



Volume:

On the TI-83, you can graph the shaded region (above) but it is difficult to visualize the rotation about the axis. You can graph reflections of your regions but it may not look nice.

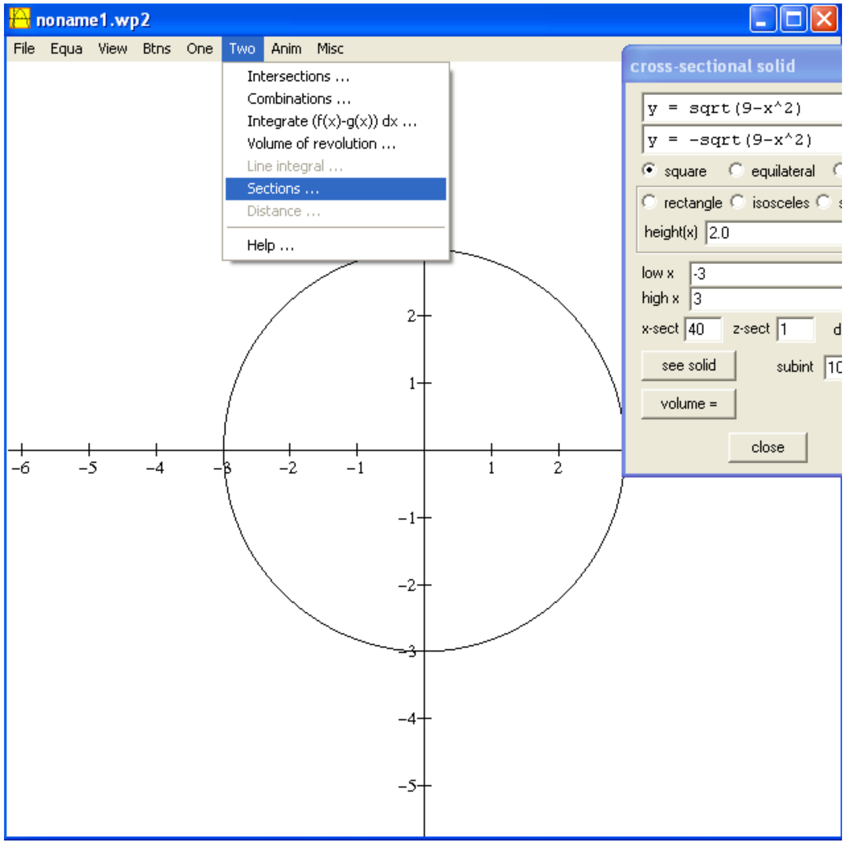
To calculate volume, use Math – 9 for fnInt

Example:

using the washer method
have functions entered into y=
enter: fnInt(π*(y1²-y2²),X,start,end)
where y1 is upper function

Lets try: Find the volume of the region found by revolving the area formed by $y = x^2$ and $y = \sqrt{x}$ about the x-axis

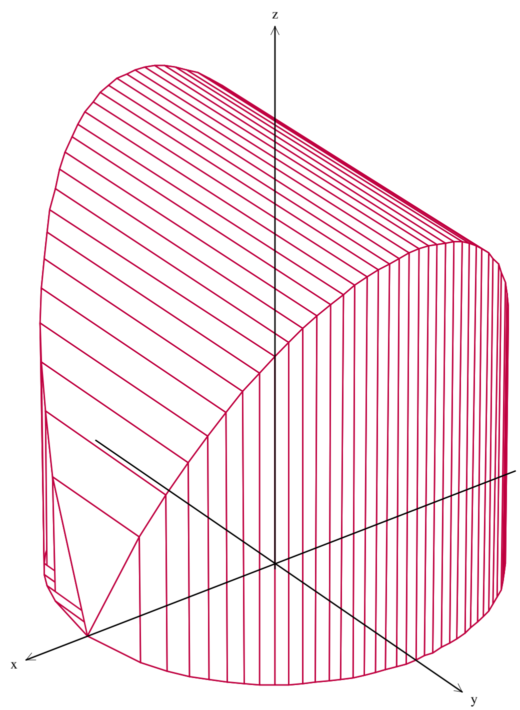
Cross sections on Winplot:



Enter base into equation(s)

Choose Two – Sections

Click see solid:



You can click volume = to see volume