The Power of Series Creation

As we learned in days gone by, the sum of an infinite geometric series is simple to calculate:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

Use this fact to create a power series for $y = \frac{1}{1-x}$.

For what values of x does this power series model the original function? Why?

How does this power series for $y = \frac{1}{1-x}$ compare the Taylor series for the function centered at $x = 0$?

How could we use the power series for $y = \frac{1}{1-x}$ to create a power series for $y = \frac{1}{1+3x}$? For what values of x is this power series valid?

Using substitution, differentiation, or integration (or any combination of those!) create eight new power series from the power series for $y = \frac{1}{1-x}$. Be creative.

	function	**power series**
example	$f(x) = \frac{1}{1-x}$	$f(x) = 1 + x + x^2 + x^3 + x^4 + \ldots$

1.

2.

3.

4.

5.

6.

7.

8.