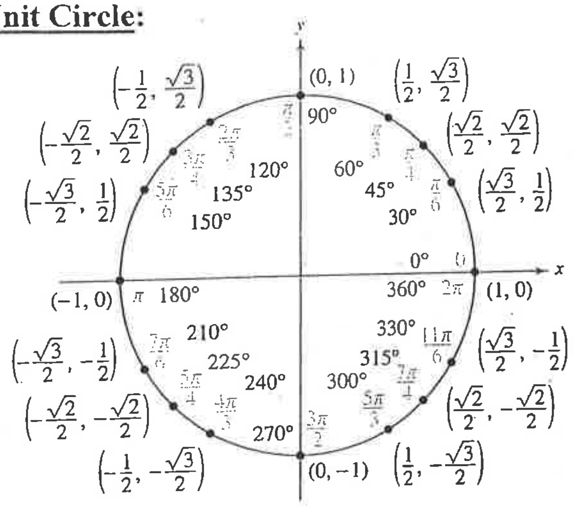
AP Calculus Formula List to Review for the AP Exam

The Unit Circle:



Few Trig Identities Useful for Integration:

$Sin2\theta = 2Sin\theta Cos\theta$

$Cos^2\theta = \dfrac{1+Cos2\theta}{2}$

$Cos2\theta = \begin{cases} Cos^2\theta - Sin^2\theta \\ 1-2Sin^2\theta \\ 2Cos^2\theta - 1 \end{cases}$

$Sin^2\theta = \dfrac{1-Cos2\theta}{2}$

$1+Tan^2\theta = Sec^2\theta$

$1+Cot^2\theta = Csc^2\theta$

Logarithmic Properties from PreCalculus :

$\ln a^n = n\ln a$ (power rule)

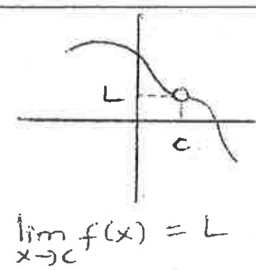
$\ln(ab) = \ln a + \ln b$

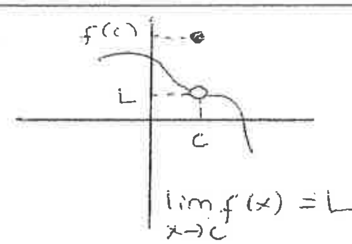
$\ln(\frac{a}{b}) = \ln a - \ln b$

$\log_c x = \dfrac{\ln x}{\ln c} = \dfrac{\log_a x}{\log_a c}$

(change of base formula)

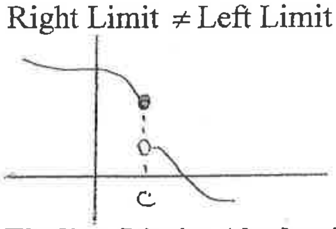
Limits: If f(x) becomes arbitrarily close to a single number L as x approaches c from either side, then the limit of f(x) as x approaches c is L



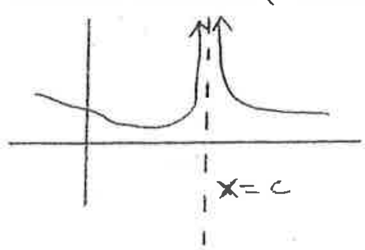


The limit of a function does not exists if

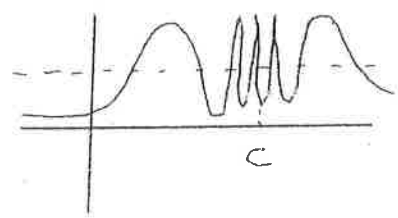
Right Limit ≠ Left Limit



Unbound Behavior (Vertical asymptote)



Oscillation



Finding Limits Algebraically

For continuous functions limit value as x approaches c and function value at c are the same. Therefore the limit of continuous functions can be found easily by substitution.

For functions with a hole (when substitution gives $\frac{0}{0}$) use 1) factoring out and simplifying

2) Rationalization (radicals)

3) Carrying out the required algebra (common denominator, add, subtract,etc)

L'Hopital's Rule for finding Limits

When substitution gives $\frac{0}{0}$ or $\frac{\infty}{\infty}$ (indeterminate forms) L'Hopital's Rule can be applied.

If $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}\ or\ \dfrac{\infty}{\infty}$ then, $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{f'(c)}{g'(c)}$ (Can be applied repetitively)

The limit of the ratio equals to the limit of the ratio of the derivatives (no quotient rule!)

Limits at Infinity: ($\lim\limits_{x\to\infty} f(x)$ is asking the horizontal (or generally end-behavior) asymptote of the function.

Power Rules from PreCalculus can be used $\dfrac{f(x)\to degree:n}{g(x)\to degree:m}$

If $n<m$ $y=0$ is the horizontal asymptote, therefore limit is 0
If $n=m$ $y=c$ (The ratio of the leading coefficients) is the horizontal asymptote, therefore limit is c
If $n>m$ No horizontal asymptote, Limit does not exist (Function values approach to ∞ or $-\infty$, not to a finite number)

Infinite Limits: If $\lim\limits_{x\to c^+} f(x)=\pm\infty$ or $\lim\limits_{x\to c^-} f(x)=\pm\infty$ then *f(x)* has a vertical asymptote at $x=c$

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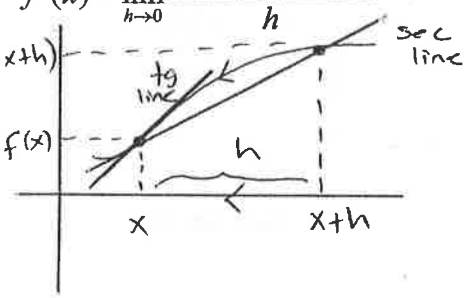
3 Special Limits: **1.** $\lim\limits_{x\to 0}\dfrac{Sin(ax)}{(ax)}=1$ **2.** $\lim\limits_{x\to 0}\dfrac{1-Cosx}{x}=0$ (**or** $\lim\limits_{x\to 0}\dfrac{Cosx-1}{x}=0$) **3.** $\lim\limits_{n\to\infty}(1+\dfrac{1}{n})^n=e$

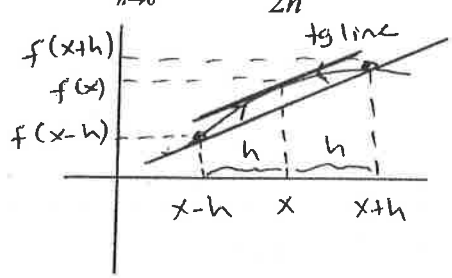
Definition of CONTINUITY: f is continuous at c iff 1. $f(c)$ is defined
2. $\lim\limits_{x\to c} f(x)$ exists
3. $f(c)=\lim\limits_{x\to c} f(x)$

Limit Definition of the Derivative:

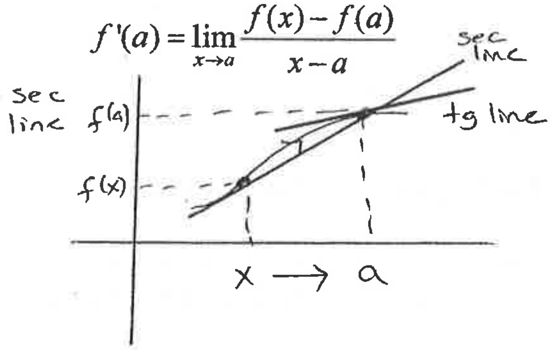
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$



$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}$$



$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

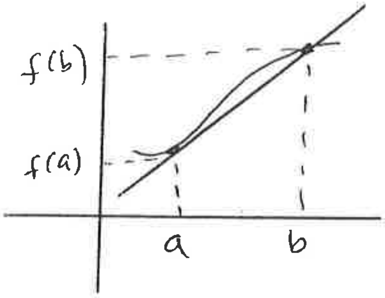


Names and Notations for the Derivative:

Slope, slope of a curve,slope of the tangent line, rate of change, instant rate of change, instantaneous rate of change, $f'(x)$, $\dfrac{dy}{dx}$, y' , $\dfrac{df(x)}{dx}$

Average Rate of Change : **(Slope of a Secant Line)**

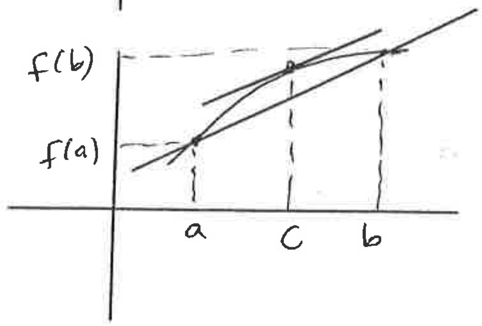
Average rate of change of $f(x)$ on [a,b] $= \dfrac{f(b)-f(a)}{b-a}$



Mean Value Theorem (MVT) :

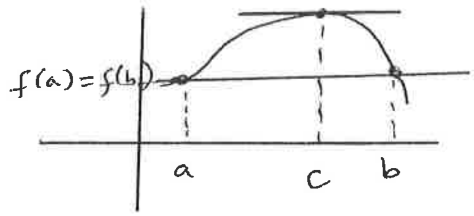
If $f(x)$ is continuous on [a,b] and differentiable on (a,b) then there exists at least one number c on (a,b) such that slope of the tangent line at c equals to the slope of the secant line passing through the points a and b

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



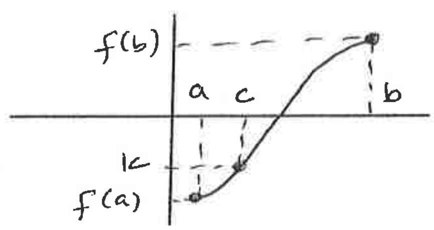
Rolle's Theorem : (Special case of MVT)

If $f(x)$ is continuous on [a,b] and differentiable on (a,b) and if $f(a) = f(b)$ then there exists at least one number c on (a,b) such that $f'(c) = 0$



Intermediate Value Theorem:

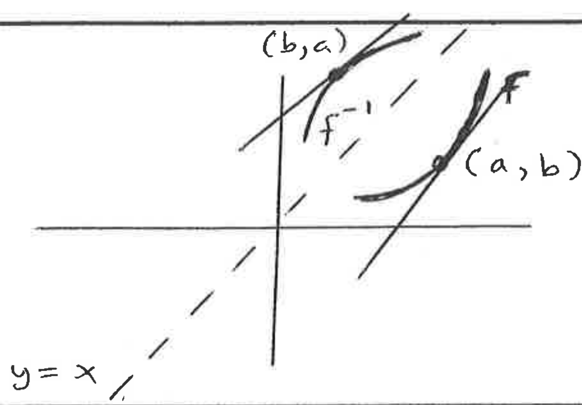
If f is continuous on [a,b] and k is any number between $f(a)$ and $f(b)$, then there is at least one number c on (a,b) such that $f(c) = k$



Derivative of the Inverse of a Function:

Point on f	Point on f^{-1}
(a,b)	(b,a)

$$f^{-1}(b)' = \frac{1}{f'(a)}$$



DIFFERENTIATION RULES

$\dfrac{d}{dx}[c] = 0$ $\qquad \dfrac{d}{dx}[x^n] = nx^{n-1}$ (power) $\qquad \dfrac{d}{dx}[uv] = uv' + vu'$ (product) $\qquad \dfrac{d}{dx}[\dfrac{u}{v}] = \dfrac{vu' - uv'}{v^2}$ (quotient)

$\dfrac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ **or** $\dfrac{d}{dx}[f(u)] = f'(u) \cdot u'$ *Chain Rule*

$$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a}\frac{du}{dx}$$

$$\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$$

$$\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arccos u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\cot u] = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\sec u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc}\csc u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

INTEGRATION RULES

$$\int dx = x + c$$

$$\int k dx = kx + c$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int e^u du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

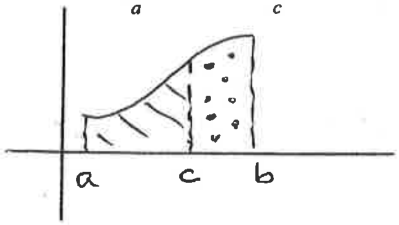
$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arc}\sec\frac{|u|}{a} + C$$

Properties of Definite Integrals:

$$\int_a^b f(x)dx = \int_a^c f(x)dx \pm \int_c^b f(x)dx$$

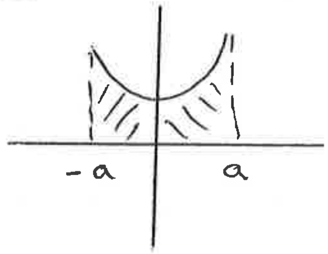


$$\int_a^a f(x)dx = 0 \qquad \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

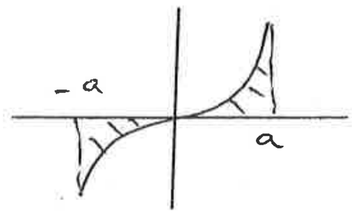
Even Functions :

$$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$$

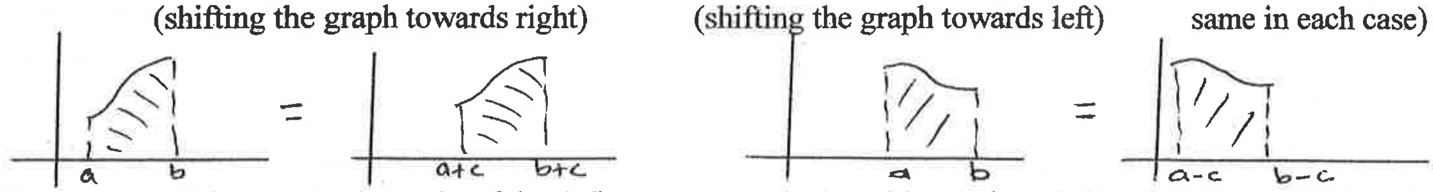


Odd Functions :

$$\int_{-a}^{a} f(x)dx = 0$$



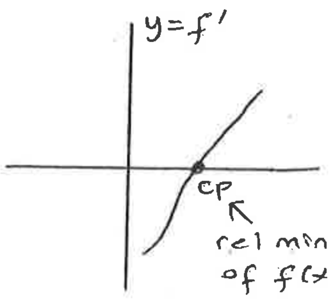
Any Function : $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$ (shifting the graph towards right) $\qquad \int_a^b f(x)dx = \int_{a-c}^{b-c} f(x+c)dx$ (shifting the graph towards left) $\rightarrow$ (Area remains the same in each case)

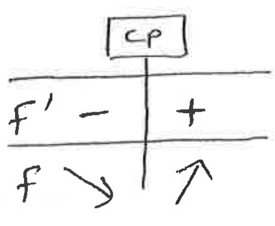


Definition of a Critical Number : Let f be defined at c . "c" is the critical point of f if $f'(c) = 0$ or $f'(c) = \varnothing$

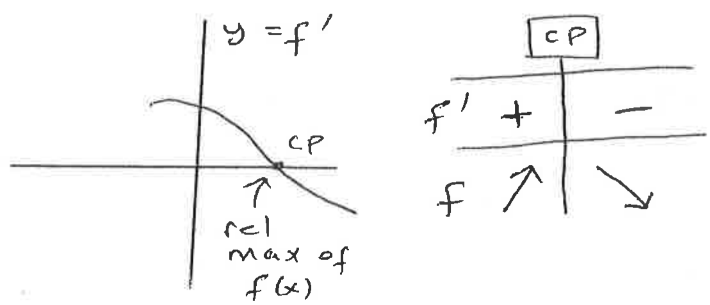
1st Derivative Test : (To find the relative extrema)
Let c be the critical point of the function that is continuous on an open interval containing c.

$f(c)$ is a relative min. of f if f': $(-) \rightarrow (+)$



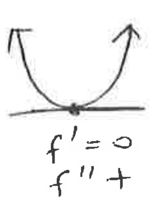


$f(c)$ is a relative max. of f if f': $(+) \rightarrow (-)$

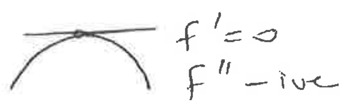


2nd Derivative Test : (To find the relative extrema)
Let f be a function such that $f'(c) = 0$ and second derivative of f exist on an open interval containing c.

$f(c)$ is a relative min. of f if $f''(c) > 0$



$f(c)$ is a relative max. of f if $f''(c) < 0$



(If $f''(c) = 0$, the 2nd derivative test is inconclusive. You need to use 1st derivative test then)

Absolute Extrema : (Extreme Value Theorem)

If function f is continuous on a closed interval [a,b], then f has a highest value (abs. max) and lowest value (abs. min) on this interval. Absolute max. or min. value of a function can be either at the end points or at the critical points ($f'=0$ or $f'=\varnothing$) of a function.

-Find the critical points - Evaluate f at the critical points and the end points - Compare the values of f and decide

Concavity :

Concave Up: f is $\cup$ - f' is $\nearrow$ - f'' is (+)ive – tangent line is below the curve of f.

Concave Down: f is $\cap$ - f' is $\searrow$ - f'' is (-)ive – tangent line is above the curve of f.

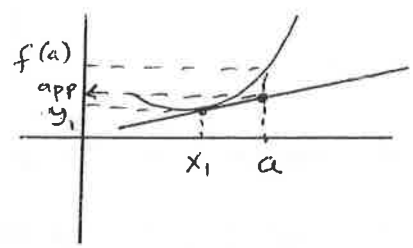
Inflection Point: A function f has an inflection point at c if $f''(c)=0$ or undefined

such that at $x=$ c f changes concavity ----f' changes from $\nearrow$ *to* $\searrow$ or vice versa -------- f'' changes sign

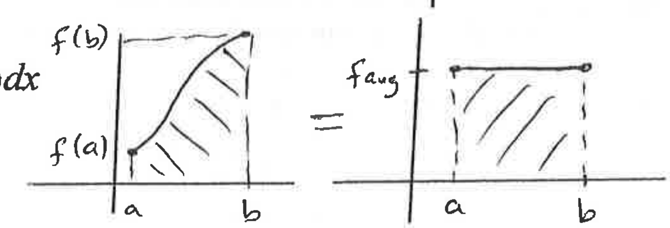
Tangent Line Approximation : (Linearization)

-Write the equation of the tangent line at the given point. $y=m_{tg}(x-x_1)+y_1$

-Plug in the x value of the nearby point into the equation ↑



Average Value of a Function : $f_{avg}=\dfrac{1}{b-a}\int_a^b f(x)dx$



FUNDAMENTAL THEOREM OF CALCULUS:

$\int_a^b f'(x)dx=f(b)-f(a)$ (FTC Part1)

$\dfrac{d}{dx}\int_c^x f(t)dt=f(x)$ (FTC Part2)

$\dfrac{d}{dx}\int_c^{g(x)} f(t)dt=f(g(x))g'(x)$ (FTC Part2-with chain rule)

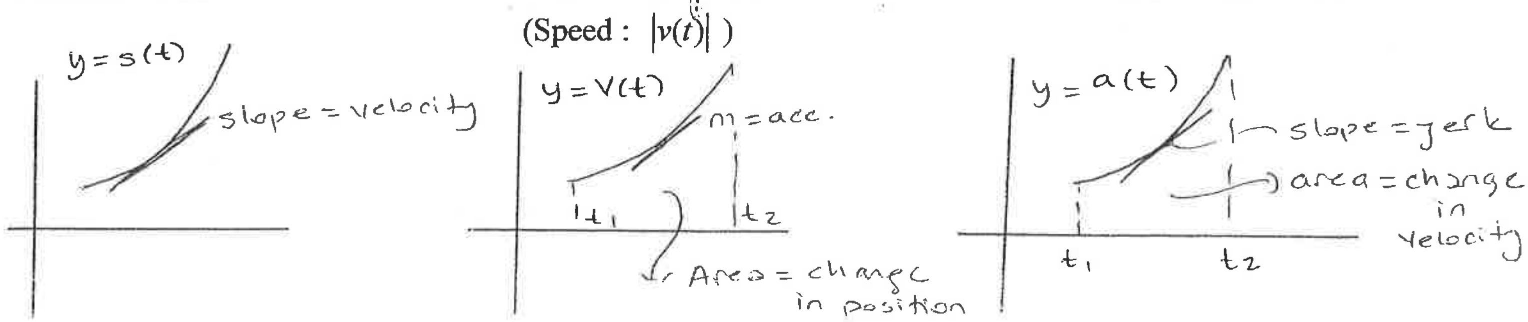
MOTION:

If an object moves along a straight line with position function s(t) , then its

Position : $s(t)$

Velocity : $v(t)=s'(t)$ (Speed : $|v(t)|$)

Acceleration : $a(t)=v'(t)=s''(t)$



When *velocity* is (+)*ive* particle moves $\rightarrow or \uparrow$, when *velocity* is (−)*ive* particle moves $\leftarrow or \downarrow$

Particle speeds up when velocity and acceleration have the same sign.

Particle slows down when velocity and acceleration have the opposite signs.

Displacement (Change in Position) from t_1 to t_2 : $\int_{t_1}^{t_2} v(t)dt$

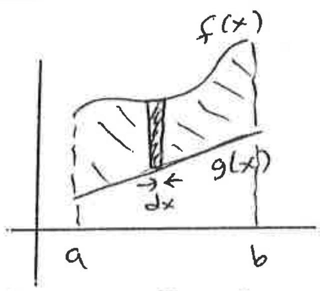
Total Distance Traveled from t_1 to t_2 : $\int_{t_1}^{t_2} |v(t)dt|$

Area Between Two Curves:

Vertical Slicing:

$$A = \int_a^b f(x) - g(x)dx$$

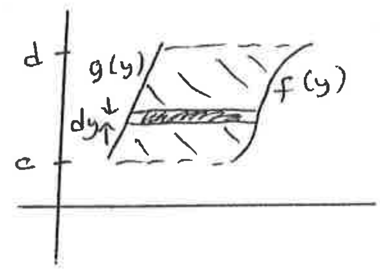
↑ upper ↑ lower



Horizontal Slicing:

$$A = \int_c^d f(y) - g(y)dy$$

↑ right ↑ left



Approximating the Area Between a Function and x-axis

On each subinterval ,
LRAM uses the y-value of the left x-value as the height of the rectangle $A = \sum f_{left} \cdot \Delta x$
MRAM uses the y-value of the middle x-value as the height of the rectangle $A = \sum f_{mid} \cdot \Delta x$
RRAM uses the y-value of the right x-value as the height of the rectangle $A = \sum f_{right} \cdot \Delta x$

TRAP uses the areas of trapezoids $A = \sum \frac{(f_{left} + f_{right})}{2} \cdot \Delta x$

Connections Between Area Approximations & Definite Integrals

As the number of subintervals increase (as $n \to \infty$ or as $\Delta x \to 0$), the error involved in each approximation decrease and we calculate the actual area

$$\lim_{n\to\infty} \sum_{i=1}^{n} f_i \cdot \Delta x_i = \int_a^b f(x)dx$$

Volumes of Rotations : (DISK or WASHER methods)

$V = \pi \int_a^b (r(x))^2 dx$ (Axis of rotation is a horizontal line) --- $V = \pi \int_c^d (r(y))^2 dy$ (Axis of rotation is a vertical line)

$V = \pi \int_a^b (r_0(x))^2 - (r_i(x))^2 dx$ (Axis of rotation is a horizontal line)

$V = \pi \int_c^d (r_o(y))^2 - (r_i(y))^2 dy$ (Axis of rotation is a vertical line)

Volumes of Solids of Known Cross Sections:

The base of the solid is the area between given curves.

Cross-sections perpendicular to x-axis (or y-axis in some prbs.) are known geometric shapes .

$V=\int_a^b A(x)dx$ or $V=\int_c^d A(y)dy$ where A is the cross sectional area written in terms of the given functions.

Separable Differential Equations :(Special case : Exponential Growth or Decay Models)

" Rate of Change of y is proportional to y " → Means $\frac{dy}{dt}=ky$

$\frac{dy}{dt}=ky$

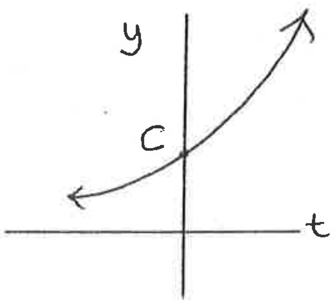
$\int \frac{1}{y}dy=\int kdt$

$\ln|y|=kt+c$

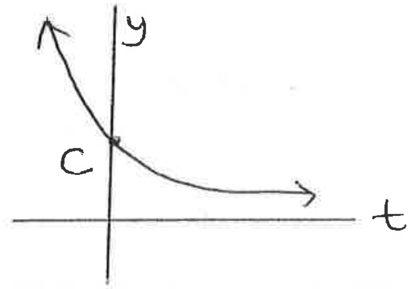
$y=e^{kt+c}=e^{kt}\cdot e^{c}$

$\boxed{y=Ce^{kt}}$

if k is (+)ive :GROWTH



if k is (-)ive :DECAY

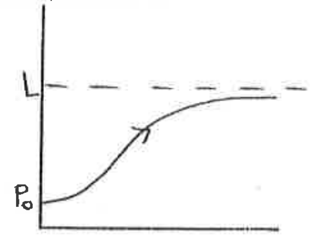


Separable Differential Equations : (Special Case : **Logistic Model**)

$$\frac{dP}{dt} = kP(L - P)$$

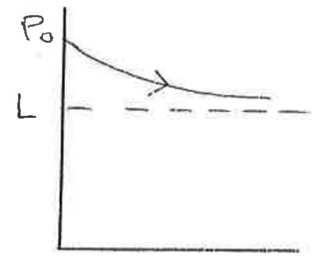
Where L is the carrying capacity.

$\lim_{t \to \infty} P(t) = L$ (no matter what the Initial population P_0)



$P < L$

$\frac{dP}{dt}$ (+)ive, $P \nearrow$



$P > L$

$\frac{dP}{dt}$ (−)ive, $P \searrow$

$\frac{dP}{dt}$ grows fastest (slope is highest) when $P = \frac{L}{2}$ because $\frac{dP}{dt} = kPL - kP^2 \rightarrow \frac{d^2P}{dt^2} = kL - 2kP = k(L - 2P) = 0$

When $L = 2P$ or $P = \frac{L}{2}$

INTEGRATION TECHNIQUES:

1) ***U-Substitution*** (indicators : there is an inner function, u, such that its derivative is also in the given integral) Try this method fist since it is the simplest one
2) ***Integration by Parts*** (indicators : the given integral is a product of any two of log., trig., polynomial, exp., or inverse trig functions)

$$\int u\,dv = uv - \int v\,du$$

↑ Complicated ↑ Simpler

Selection of ***u***

Log-**I**nv.trig-**P**olyn.-**E**xp.-**T**rig

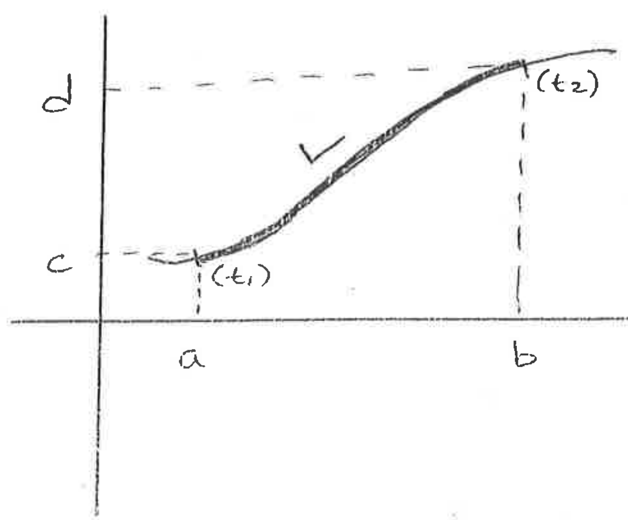
3) ***Integration by Partial Fractions*** (indicators: integral is in fraction form where denominator is factorable)

4) ***Improper Integrals*** (indicators : one or both limits of integration is ∞, or function in the integral has a vertical asymptote within the given limits of integration)

ARC LENGTH :

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

↑ w.r.to x ↑ w.r.to y



in Parametric Form : $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

PARAMETRIC FUNCTIONS

Given $x(t)$ and $y(t)$, the slope of the tangent line to the parametric curve is $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

(When $\frac{dy}{dt}$ is zero, tg. line is horizontal and when $\frac{dx}{dt}$ is zero tg. line is vertical)

Second derivative of a parametric function is $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$ (derivative of the 1st derivative / $\frac{dx}{dt}$)

VECTOR VALUED FUNCTIONS (Vectors in a plane region)

Given the position functions $x(t)$ and $y(t)$,

Position Vector: $\vec{S}(t) = \langle x(t), y(t) \rangle$ Velocity Vector: $\vec{V}(t) = \langle x'(t), y'(t) \rangle$ Acceleration Vector: $\vec{a}(t) = \langle x''(t), y''(t) \rangle$

Distance Traveled by the Particle: $L = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ (This is the arc length in parametric form)

Speed : $\sqrt{(x'(t)^2 + (y'(t)^2}$ (This is the magnitude of the velocity vector)

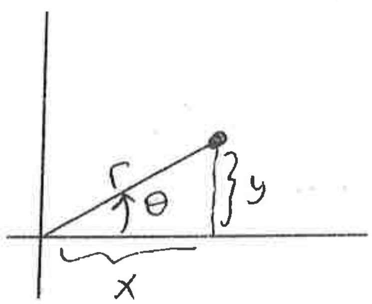
POLAR FUNCTIONS:

$x = rCos\theta$

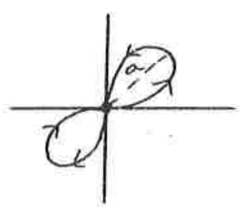
$y = rSin\theta$

$x^2 + y^2 = r^2$

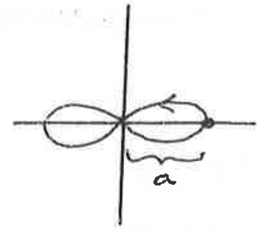
$\tan\theta = \frac{y}{x} \rightarrow \theta = \arctan\frac{y}{x}$



Two Special Polar Curves : $r^2 = a^2Sin(2\theta)$



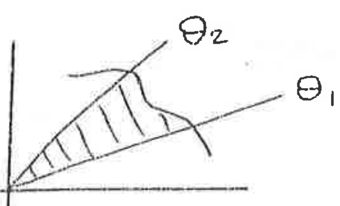
$r^2 = a^2Cos(2\theta)$



$\frac{dy}{dx}$ in Polar Form : $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{rCos\theta + r'Sin\theta}{-rSin\theta + r'Cos\theta}$ (Product rules applied in the numerator & denominator)

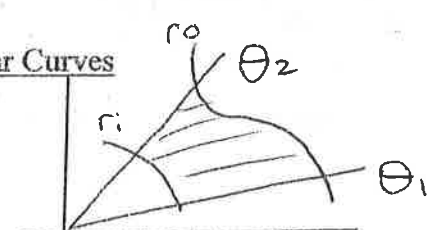
Area of a Polar Region

$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 d\theta$



Area Between Two Polar Curves

$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r_o^2 - r_i^2 d\theta$



Note : $\frac{dr}{d\theta}$ is (+)ive means $r \nearrow$ and the polar curve moves away from the origin

$\frac{dr}{d\theta}$ is (-)ive means $r \searrow$ and the polar curve moves towards the origin

TAYLOR POLYNOMIALS (centered at c)

$$T_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \ldots \frac{f^{(n)}(c)}{n!}(x-n)^2$$

MACLAURIN POLYNOMIALS (Taylor Polynomials centered at 0)

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots \frac{f^{(n)}(0)}{n!}x^2$$

Some Important Maclaurin Series to Memorize (These can be manipulated)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots\ldots x^n + \ldots = \sum_{n=0}^{\infty} x^n$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$Sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$Cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$ This can also be obtained by integrating *Sinx* series

GEOMETRIC SERIES

$$a + ar + ar^2 + ar^3 + \ldots\ldots + ar^n + \ldots = \sum_{n=0}^{\infty} ar^n = \sum_{n=1}^{\infty} ar^{n-1}$$

If $|r| < 1$ **the series is convergent (i.e. the sum exists), The Sum is** $S = \frac{a}{1-r}$

If $|r| \geq 1$ **the series is divergent (i.e. the sum does not exists)**

Geometric series are the only series for which we have the formula for the sum. For all other series we can only calculate an interval for the sum using the error formulas.

Series Convergence Tests

n-th Term Test	$\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow \lim_{n\to\infty} a_n = 0$. $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.
Integral Test	$f(x)$ is continuous, positive, and decreasing. $\sum_{n=1}^{\infty} f(n)$ converges $\Leftrightarrow \int_M^{\infty} f(x)\,dx$ converges (for some M).
p-Series—	$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges $\Leftrightarrow p > 1$.
Comparison Test	$0 < a_n < b_n$. $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges. $\sum_{n=1}^{\infty} a_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} b_n$ diverges.
Ratio Test	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ converges. $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges. $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1 \Rightarrow$ can't tell.
Alternating Series Test	$a_n > 0$, decreasing, $\lim_{n\to\infty} a_n = 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges.

SERIES AND ERROR

The AP Calculus BC course description includes two kinds of error bounds:

- Alternating series with error bound
- Lagrange error bound for Taylor polynomials

Both types of error have been tested on the AP test.

sum = $\underbrace{a_1 + a_2 + \cdots a_n}_{S_n} + R_n$ ← ERROR

Alternating Series Remainder

Suppose an alternating series satisfies the conditions of the Alternating Series Test: namely, that $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}$ is a decreasing sequence $(a_{n+1} < a_n)$. If the series has a sum S, then $|R_n| = |S - S_n| \le a_{n+1}$, where S_n is the nth partial sum of the series.

In other words, if an alternating series satisfies the conditions of the Alternating Series Test, you can approximate the sum of the series by using the nth partial sum, S_n, and your error will have an absolute value no greater than the first term left off, a_{n+1}.

$T(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x) = \text{sum}$

$R_n(x)$: Error by using first n terms

Taylor's Inequality

Suppose that $P_n(x)$ is the nth-degree polynomial approximation for the function f about $x = c$ and M is the maximum value of $\left|f^{(n+1)}(x)\right|$ on the interval $[c, b]$ (or $[b, c]$ if $b < c$). Then the error in using the polynomial value $P_n(b)$ to estimate $f(b)$ is bounded by $\frac{M}{(n+1)!}|b-c|^{n+1}$.

That is, the remainder $R_n(x)$ in Taylor's Formula satisfies the inequality

$$|R_n(x)| \le \left|\frac{M}{(n+1)!}(b-c)^{n+1}\right|.$$

Version B

Derivative Practice

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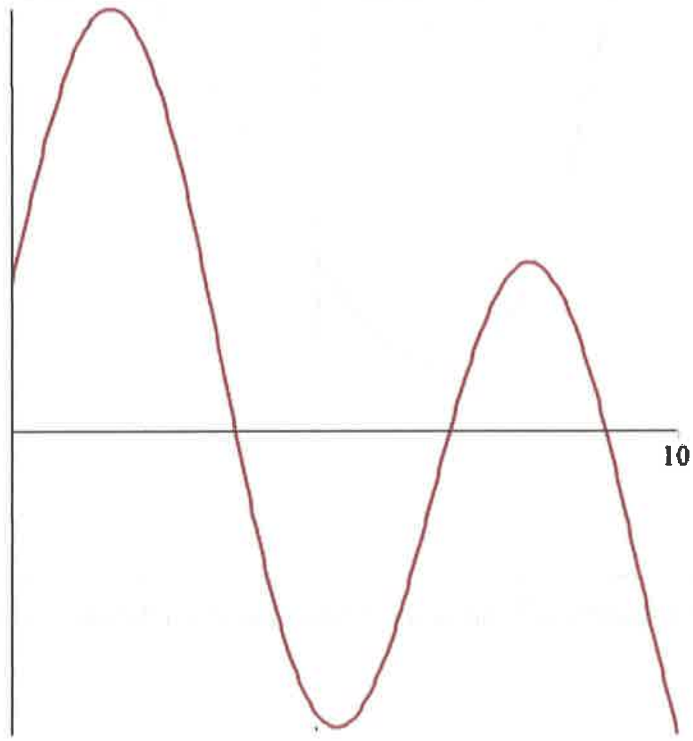
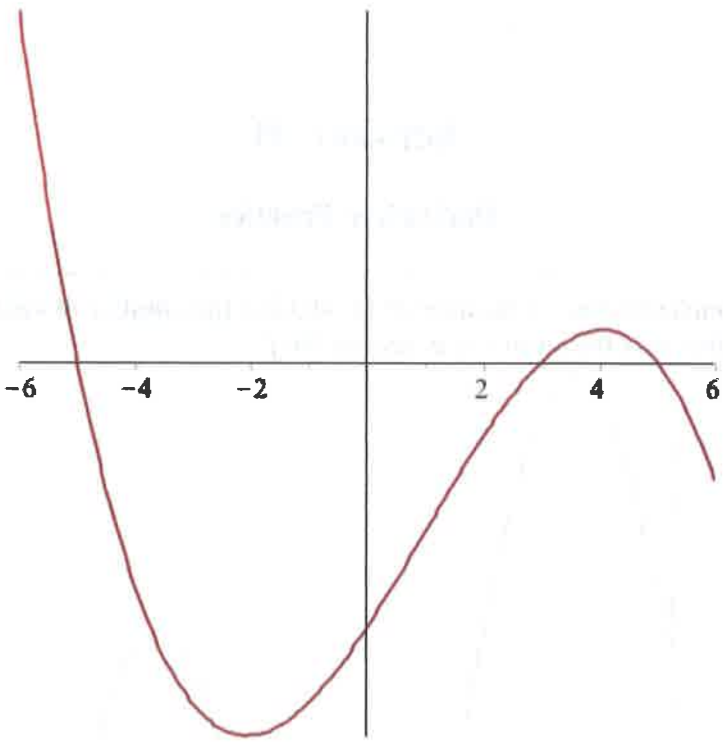
1) The function f is graphed below on the interval [0,10].Give the **number of values** c between 0 and 10 which satisfy the conclusion of the mean value theorem for f.



a) 4
b) 3
c) 1
d) 2
e) 5

---

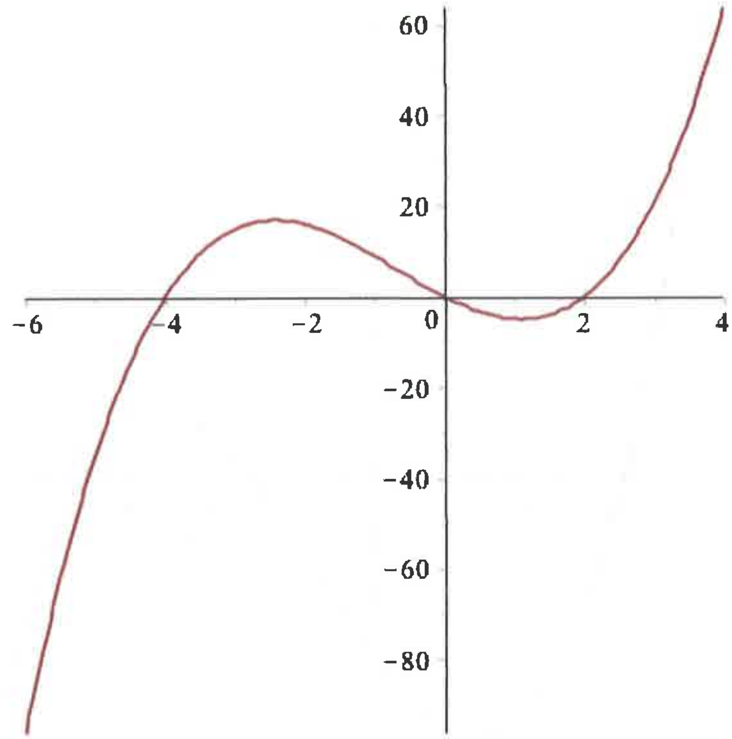
2) Suppose that $c = 3$ is a critical number for a function f. Determine if $f(c)$ is a local maximum, local minimum or neither if the graph of $f'(x)$ is shown below.



a) Local Minimum
b) Neither
c) Local Maximum

---

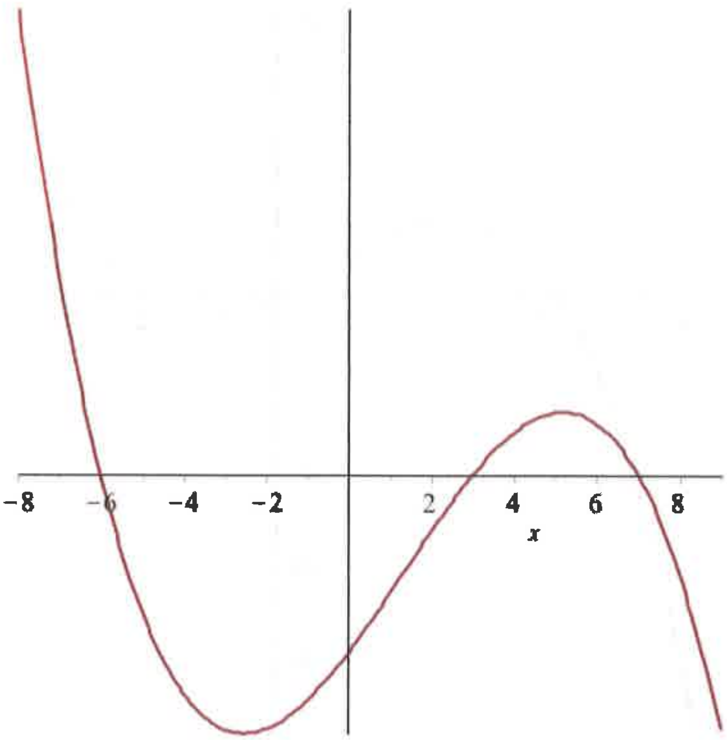
3) **Read Carefully!** The graph of f' (the derivative of f) is shown below. Classify the smallest critical number for f.



a) local minimum
b) neither
c) local maximum

---

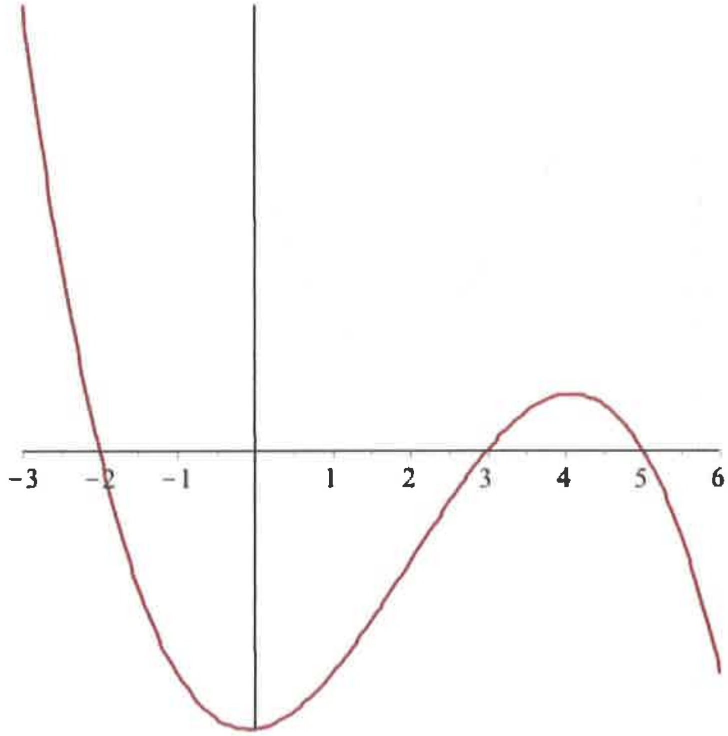
4) Given the graph of $f'(x)$ below, where is $f(x)$ decreasing?



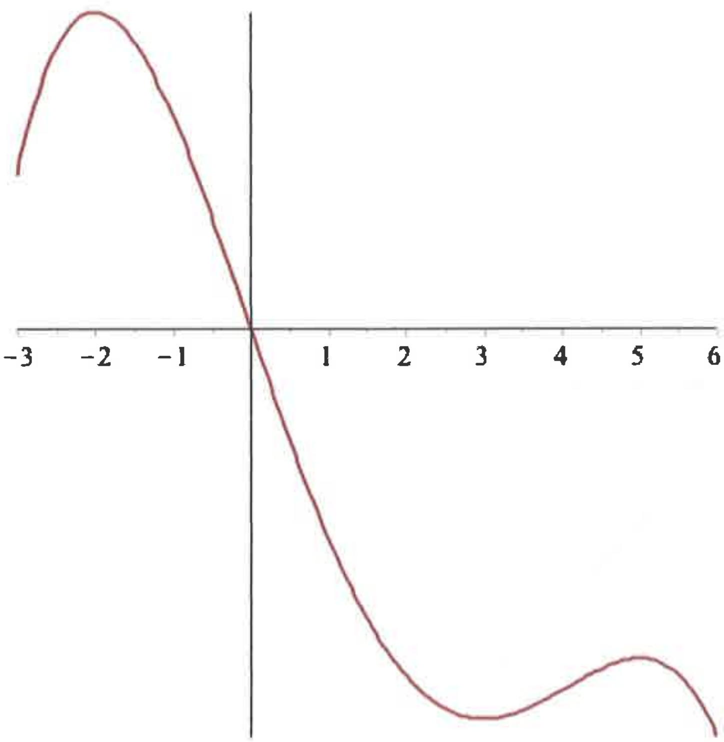
a) $f(x)$ is decreasing on the interval $(-6, 7)$.
b) $f(x)$ is decreasing on the intervals $(-6, 3)$ and $(7, \infty)$.
c) $f(x)$ is decreasing on the interval $(-6, \infty)$.
d) $f(x)$ is decreasing on the intervals $(-\infty, -6)$ and $(3, 7)$.
e) $f(x)$ is decreasing on the interval $(-\infty, 7)$.

---

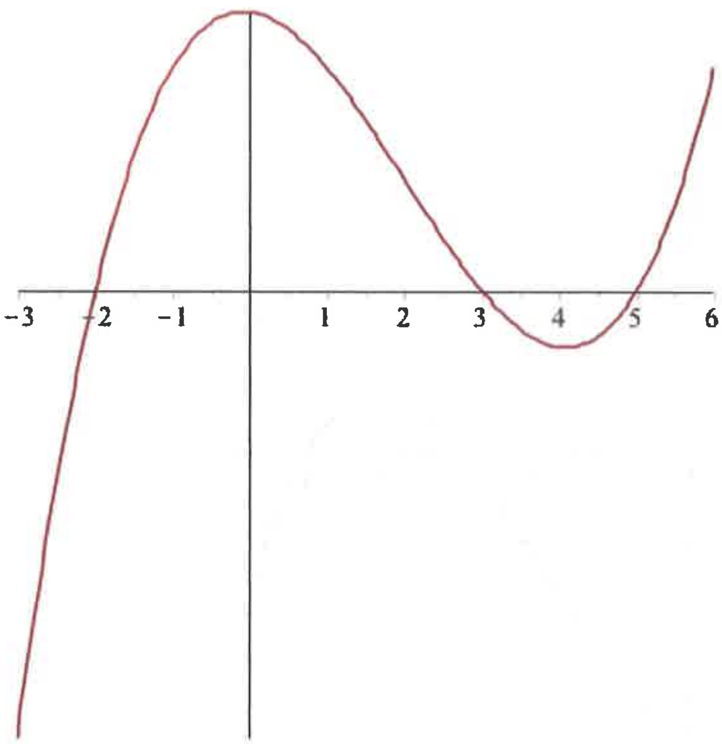
5) The graph of $f'(x)$ is shown below. Which of the following could represent the graph of $f(x)$?



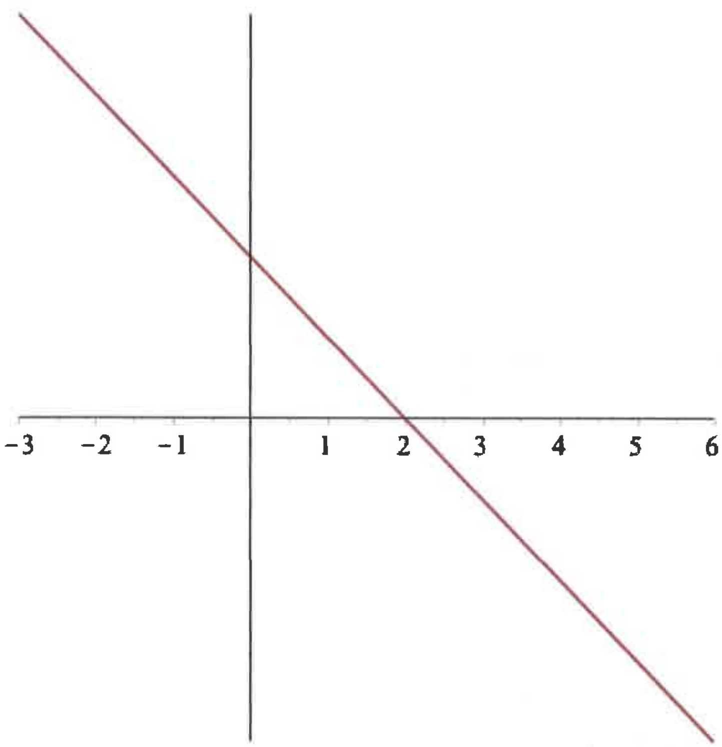
a)

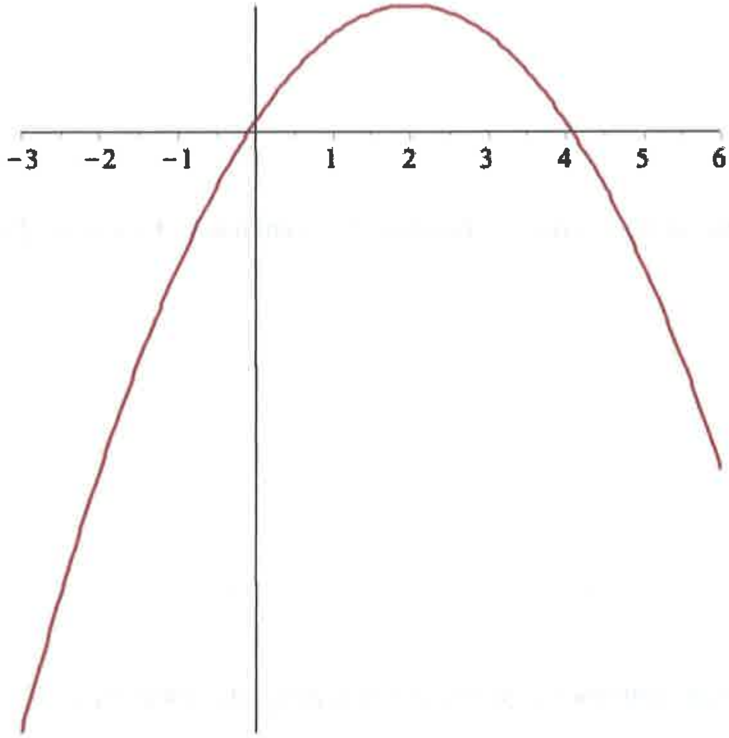


b)



c)



d) 

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6) Given $f(x) = 6x^2 - 3\sqrt{x}$ which of the following expressions will represent $f'(x)$?

a) $\lim_{h\to 0} \frac{\left(6x^2 - 3\sqrt{x} + h\right) - \left(6x^2 - 3\sqrt{x}\right)}{h}$

b) $\frac{\left(6(x+h)^2 - 3\sqrt{x+h}\right) - \left(6x^2 - 3\sqrt{x}\right)}{h}$

c) $\lim_{h\to 0} \frac{\left(6(x+h)^2 - 3\sqrt{x+h}\right) - \left(6x^2 - 3\sqrt{x}\right)}{h}$

d) $\lim_{h\to x} \frac{\left(6(x+h)^2 - 3\sqrt{x+h}\right) - \left(6x^2 - 3\sqrt{x}\right)}{h}$

e) $\lim_{h\to 0} \frac{6(x+h)^2 - 3\sqrt{x+h}}{h}$

---

7) $\lim_{h\to 0} \frac{\frac{1}{9+h} - \frac{1}{9}}{h} =$

a) $-\frac{1}{81}$

b) $\frac{1}{9}$

c) does not exist

d) 0

e) $-\frac{1}{9}$

---

8) The limit $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$ represents the derivative of a function f at a number c. Determine f and c.

a) $f(x) = (2+x)^2, c = -2$
b) $f(x) = x^2, c = 2$
c) $f(x) = (2+x)^2, c = 2$
d) $f(x) = (2-x)^2, c = 4$
e) $f(x) = x^2, c = 4$

---

9) The limit $\lim_{h \to 0} \frac{\cot\left(\frac{\pi}{3} + h\right) - \frac{\sqrt{3}}{3}}{h}$ represents the derivative of a function f at a number c. Determine f and c.

a) $f(x) = \cot(x), c = \frac{\pi}{3}$

b) $f(x) = -\cot(x), c = \frac{\sqrt{3}}{3}$

c) $f(x) = \cot(1/3\,\pi x), c = \frac{\sqrt{3}}{3}$

d) $f(x) = \cot(x), c = \frac{\sqrt{3}}{3}$

e) $f(x) = \cot(1/3\,\pi x), c = \frac{\pi}{3}$

---

10)
The functions f and g are differentiable and $h(x) = f(g(x))$. Use the information below to find $h'(0)$.

$$\begin{array}{cc} f(0) = -2 & f'(0) = 3 \\ f(2) = -5 & f'(2) = 4 \\ g(0) = 2 & g'(0) = -2 \\ g(-5) = 5 & g'(-5) = -3 \end{array}$$

a) 8
b) – 8

c) 0
d) – 16
e) 2

---

11) Express the deriviative $\frac{d}{dx}\left(f\left(2x^2+5\right)\right)$ in terms of f'.

a) $f'\,(4x)$
b) $4x \cdot f'\left(2x^2+5\right)$
c) $4x \cdot f\left(2x^2+5\right) \cdot f'(x)$
d) $f'\left(2x^2+5\right)$
e) $4x$

Calculus Applets

Limits and Continuity
Lesson 2 and 5 – other calculus topics here
www.calculus-help.com/tutorials

Continuity & epsilon/delta game
http://www.austincc.edu/powens/HTMLJava/contin/continuity.htm

Graphing derivative
http://www.ltcconline.net/greenl/java/other/derivativegraph/classes/derivativegraph.html

Related Rates
Number 23 -- related rates using GeoGebra -- other calculus demonstrations here
http://webspace.ship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplets.html

Estimating Distance Traveled from Velocity Curves
Section 5.3 example -- calculus examples for Hughes-Hallett text
http://higheredbcs.wiley.com/legacy/college/mccallum/0470131586/applets/HHapplets.htm

Volume
Number 27 Volumes I
http://online.math.uh.edu/HoustonACT/videocalculus/
Section 8.2 example 4 – calculus examples or Hughes-Hallett text
http://higheredbcs.wiley.com/legacy/college/mccallum/0470131586/applets/HHapplets.htm

Volume – Shell Method
http://archives.math.utk.edu/visual.calculus/5/volumes.6/index.html
Number 29 Volumes III
http://online.math.uh.edu/HoustonACT/videocalculus/

Alternating Series and Absolute Convergence
Number 52 slides 1-10
http://online.math.uh.edu/HoustonACT/videocalculus/

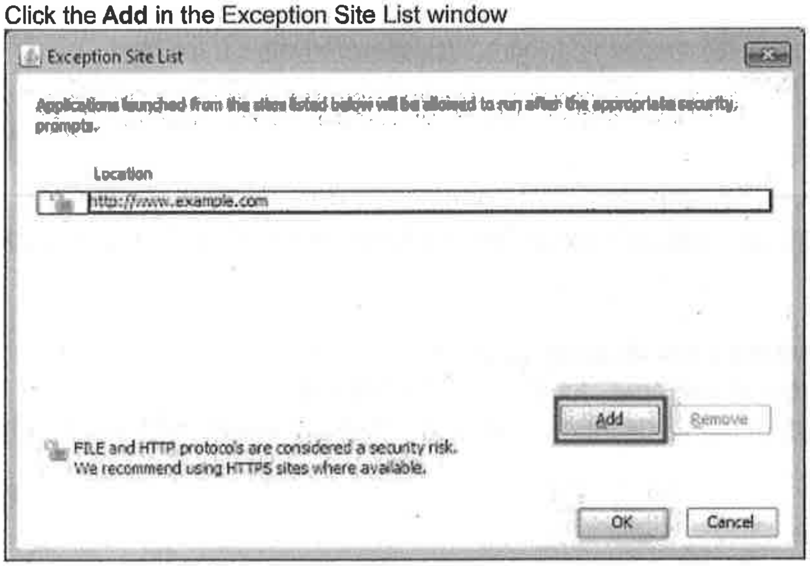
Why are Java applications blocked by your security settings with the latest Java?
http://java.com/en/download/help/java_blocked.xml

WORKAROUND

It is highly recommended not to run these kind of applications, however if you still want to run, run only if you understand risk and implications involved.
As a workaround, the user can use **Exception Site list** feature to run the applications blocked by security settings. By adding the URL of the blocked application to the Exception Site list allows it to run with some warnings.

Steps to Add URL to the Exception Site list:

- Go to the **Java Control Panel** (On Windows Click Start, All Programs, Java and then Configure Java)
- Click on the **Security** tab
- Click on the **Edit Site List** button
- Click the **Add** in the Exception Site List window



- Click in the empty field under Location field to enter the URL
  Example: http://higheredbcs.wiley.com

- Click OK to save the URL that you entered.

Click Continue on the Security Warning dialog