AP Calculus Mock Exam

AB 2

t	0	2	6	8	10	12
$y'(t)$	4	8	-2	3	-1	-5

The vertical position of a particle moving along the y-axis is modeled by a twice-differentiable function $y(t)$ where t is measured in seconds and $y(t)$ is measured in meters. Selected values of $y'(t)$, the derivative of $y(t)$, over the interval $0 \le t \le 12$ seconds are shown in the table above. The position of the particle at time $t = 12$ is $y(12) = -3$.

(a) Use a locally linear approximation of y at $t = 12$ to approximate $y(11.8)$.

(b) Approximate $y''(4)$ using the average rate of change of $y'(t)$ on the interval $2 \le t \le 6$.

(c) Using correct units, explain the meaning of $y''(4)$ in the context of the problem.

(d) Find the average value of the acceleration of the particle over the interval $[0, 12]$.

(e) Using a midpoint Riemann sum and three subintervals of equal length, approximate $\int_0^{12} y'(t)\,dt$.

(f) Using correct units, explain the meaning of $\int_0^{12} y'(t)\,dt$ in the context of the problem.

(g) Explain why there must be at least three times t in the interval $0 < t < 12$ such that $y'(t) = 0$.

(h) Explain why there must be at least two times t in the interval $0 < t < 12$ such that $y''(t) = 0$.

Solution	Scoring
(a) $y'(12) = -5$ An equation of the tangent line is $y = (-5)(x - 12) - 3$ $y(11.8) \approx (-5)(11.8 - 12) - 3 = -2$	2: $\begin{cases} 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$
(b) $y''(4) \approx \dfrac{y'(6) - y'(2)}{6 - 2} = \dfrac{-2 - 8}{4} = -2.5$	1 : approximation
(c) $y''(4)$ is the acceleration of the particle in meters per second per second at time $t = 4$ seconds.	1 : meaning with units
(d) $\dfrac{1}{12}\displaystyle\int_0^{12} y''(t)\,dt = \dfrac{1}{12}\Big[y'(t)\Big]_0^{12}$ $= \dfrac{1}{12}[y'(12) - y'(0)]$ $= \dfrac{1}{12}[-5 - 4] = -\dfrac{3}{4}$	3: $\begin{cases} 1 : \text{integral} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$
(e) $\displaystyle\int_0^{12} y'(t)\,dt$ $\approx y'(2) \cdot (4 - 0) + y'(6) \cdot (8 - 4) + y'(10) \cdot (12 - 8)$ $= (8)(4) + (-2)(4) + (-1)(4) = 20$	2: $\begin{cases} 1 : \text{midpoint Riemann sum} \\ 1 : \text{estimate} \end{cases}$
(f) $\displaystyle\int_0^{12} y'(t)\,dt$ is the vertical displacement (change in position), in meters, of the particle over the time interval $0 \le t \le 12$ seconds.	2: $\begin{cases} 1 : \text{interpretation} \\ 1 : \text{units} \end{cases}$
(g) y is twice differentiable $\Rightarrow y'$ is differentiable $\Rightarrow y'$ is continuous over the interval $[0, 12]$. $y'(t)$ changes from positive to negative on the interval $[2, 6]$. $y'(t)$ changes from negative to positive on the interval $[6, 8]$. $y'(t)$ changes from positive to negative on the interval $[8, 10]$. By the Intermediate Value Theorem, there must be times a, b, and c, such that $2 < a < 6 < b < 8 < c < 10$, and $y'(a) = y'(b) = y'(c) = 0$	2: $\begin{cases} 1 : \text{conditions} \\ 1 : \text{conclusion using the Intermediate Value Theorem} \end{cases}$

Solution	Scoring
(h) y is twice differentiable $\Rightarrow y'$ is differentiable $\Rightarrow y'$ is continuous over the interval $[0, 12]$. From part (g): there are times $t = a$, $t = b$, and $t = c$ such that $0 < a < b < c < 12$ and $y'(a) = y'(b) = y'(c) = 0$. $\dfrac{y'(b) - y'(a)}{b - a} = 0$ and $\dfrac{y'(c) - y'(b)}{c - b} = 0$ By the Mean Value Theorem (or Rolle's Theorem) there must be a time $t = t_1$ such that $a < t_1 < b$ and $y''(t_1) = 0$ and a time $t = t_2$ such that $b < t_2 < c$ and $y''(t_2) = 0$	2: { 1 : conditions 1 : conclusion using the Mean Value Theorem