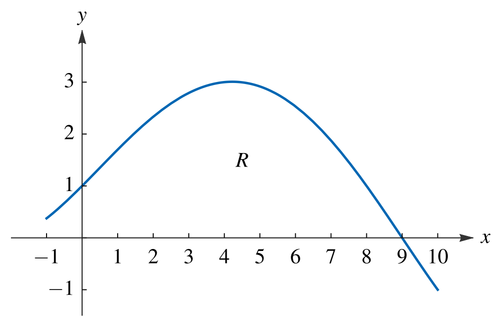
AP Calculus Mock Exam

BC 1

The graph of g', the derivative of the twice-differentiable function g, is shown for $-1 < x < 10$. The graph of g' has exactly one horizontal tangent line, at $x = 4.2$.



Graph of g'

Let R be the region in the first quadrant bounded by the graph of g' and the x-axis from $x = 0$ to $x = 9$. It is known that $g(0) = -7$, $g(9) = 12$, and $\int_0^9 g(x)\,dx = 27.6$.

(a) Find all values of x in the interval $-1 < x < 10$, if any, at which g has a critical point. Classify each critical point as the location of a relative minimum, relative maximum, or neither, Justify your answers.

(b) How many points of inflection does the graph of g have on the interval $-1 < x < 10$? Give a reason for your answer.

(c) Find the area of the region R.

(d) Write an expression that represents the perimeter of the region R. Do not evaluate this expression.

(e) Must there exist a value of c, for $0 < c < 9$, such that $g(c) = 0$? Justify your answer.

(f) Evaluate $\int_0^9 \left[\frac{1}{2}g(x) - \sqrt{x}\right] dx$. Show the computations that lead to your answer.

(g) Evaluate $\lim_{x \to 0} \frac{x \cos x}{g(x) + 2x + 7}$. Show the computations that lead to your answer.

(h) Let h be the function defined by $h(x) = \int_{x^2}^{0} g(t)\,dt$. Find $h'(3)$. Show the computations that lead to your answer.

(i) The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis is a right triangle with height x and base in the region R. The volume of the solid is given by $\int_0^9 A(x)\,dx$. Write an expression for $A(x)$.

(j) Find the volume of the solid described in part (h). Show the computations that lead to your answer.

(k) Find the value of $\int_0^9 \frac{g''(x)}{g'(x)}\,dx$ or show that it does not exist.

(l) If $g''(0) = 0.7$, find the second degree Taylor polynomial for g about $x = 0$.

Solution	Scoring
(a) $g'(x) = 0$: $x = 9$ $g'(x)$ DNE: none g has a critical point at $x = 9$. At $x = 9$, g has a relative maximum because $g'(x)$ changes from positive to negative there.	2: { 1 : critical point at $x = 9$ 1 : relative maximum with justification
(b) The graph of g has a point of inflection where g' changes from increasing to decreasing or from decreasing to increasing. g' changes from increasing to decreasing at $x = 4.2$. Therefore the graph of g has one point of inflection at the point where $x = 4.2$.	2: { 1 : answer 1 : reason
(c) Area $= \int_0^9 g'(x)\,dx = \Big[g(x)\Big]_0^9$ $= g(9) - g(0) = 12 - (-7) = 19$	3: { 1 : definite integral for area 1 : Fundamental Theorem of Calculus 1 : answer
(d) $P = 1 + 9 + \int_0^9 \sqrt{1 + g''(x)^2}\,dx$	2: { 1 : definite integral 1 : answer
(e) Since g is differentiable, then g is continuous on $0 \le x \le 9$. $g(0) = -7 < 0 < 12 = g(9)$ By the Intermediate Value Theorem, there exists a value of c, for $0 < c < 9$, such that $g(c) = 0$.	2: { 1 : conditions 1 : conclusion using the Intermediate Value Theorem
(f) $\int_0^9 \left[\frac{1}{2}g(x) - \sqrt{x}\right] dx = \frac{1}{2}\int_0^9 g(x)\,dx - \int_0^9 \sqrt{x}\,dx$ $= \frac{1}{2}(27.6) - \left[\frac{2}{3}x^{3/2}\right]_0^9$ $= 13.8 - \frac{2}{3}(27)$ $= 13.8 - 18 = -4.2$	3: { 1 : properties of definite integrals 1 : antiderivative of $\sqrt{x}$ 1 : answer

Solution	Scoring
(g) $\lim_{x\to 0}(x\cos x) = 0$ $\lim_{x\to 0}(g(x) + 2x + 7) = 0$ Therefore the limit $\lim_{x\to 0} \frac{x\cos x}{g(x)+2x+7}$ is in the indeterminate form $\frac{0}{0}$ and L'Hospital's Rule can be applied. $\lim_{x\to 0} \frac{x\cos x}{g(x)+2x+7} = \lim_{x\to 0} \frac{x\cdot(-\sin x) + 1\cdot\cos x}{g'(x)+2}$ $= \frac{0\cdot(-\sin 0) + 1\cdot\cos 0}{g'(0)+2} = \frac{1}{3}$	3 : { 1 : conditions for L'Hospital's Rule 1 : applies L'Hospital's Rule 1 : answer
(h) $h'(x) = \frac{d}{dx}\left[\int_{x^2}^{0} g(t)\,dx\right]$ $= -\frac{d}{dx}\left[\int_{0}^{x^2} g(t)\,dt\right]$ $= -g(x^2)\cdot(2x) = -2xg(x^2)$ $h'(3) = -2\cdot 3\cdot g(9) = -6\cdot 12 = -72$	3 : { 1 : Fundamental Theorem of Calculus 1 : Chain Rule 1 : answer
(i) $A(x)$ represents the area of a right triangle at each x. $A(x) = \frac{1}{2}xg'(x)$	1 : answer
(j) $V = \int_0^9 A(x)\,dx = \frac{1}{2}\int_0^9 xg'(x)\,dx$ Use integration by parts. $u = x \qquad dv = g'(x)\,dx$ $du = dx \qquad v = \int g'(x)\,dx = g(x)$ $V = \frac{1}{2}\left(\Big[x\cdot g(x)\Big]_0^9 - \int_0^9 g(x)\,dx\right)$ $= \frac{1}{2}([9\cdot g(9) - 0\cdot g(0)] - 27.6)$ $= \frac{1}{2}(9\cdot 12 - 27.6) = 40.2$	2 : { 1 : integration by parts 1 : answer

Solution	Scoring
(k) $\int_0^9 \frac{g''(x)}{g'(x)}\,dx = \lim_{t\to 9^-}\int_0^t \frac{g''(x)}{g'(x)}\,dx$ Let $u = g'(x)$, then $du = g''(x)\,dx$ and $dx = \frac{du}{g''(x)}$ $\int \frac{g''(x)}{g'(x)}\,dx = \int \frac{g''(x)}{u}\cdot\frac{du}{g''(x)} = \int \frac{du}{u}$ $= \ln\lvert u\rvert = \ln\lvert g'(x)\rvert$ $\lim_{t\to 9^-}\int_0^9 \frac{g''(x)}{g'(x)}\,dx = \lim_{t\to 9^-}\Big[\ln g'(x)\Big]_0^t$ $= \lim_{t\to 9^-}[\ln g'(t) - \ln g'(0)]$ $= \lim_{t\to 9^-}\ln g'(t) = -\infty$ Therefore the improper integral does not exist.	3 : { 1 : improper integral 1 : antiderivative 1 : answer
(l) $g(0) = -7,\quad g'(0) = 1,\quad g''(0) = 0.7$ $T_2(x) = g(0) + g'(0)x + \frac{g''(0)}{2!}x^2$ $= -7 + 1\cdot x + \frac{0.7}{2}\,x^2$ $= -7 + x + 0.35\,x^2$	2 : { 1 : form of $T_2(x)$ 1 : answer