Differential Equations from Multiple Perspectives using Winplot
HoustonACT
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- Introduction to Winplot

  - Winplot is available at http://math.exeter.edu/rparris/winplot.html. It is worth learning to downloading winplot because there are frequent updates.
  - The website http://spot.pcc.edu/~ssimonds/winplot/ gives detailed user instruction in an animated format for winplot. I highly recommend working through the videos if you want to become more familiar with winplot.
  - Basics with winplot.

- Introducing Differential Equations.

  - Decide whether $y = \frac{x^2}{2}$ is a solution to the differential equation $\frac{dy}{dx} = x$.
  - Decide whether $y = 3x^3 + 5$ is a solution to the differential equation $\frac{dy}{dx} = 9x^2 + 1$.
  - Determine the value of A so that $y = Ax^2$ is a solution to the differential equation $\frac{dy}{dx} = 3x$
  - Find all value of p so that $y = x^p$ is a solution of the differential equation $3x\frac{dy}{dx} = y$
  - Find all values of B so that $y = \sqrt{Bx+3}$ is a solution to $\frac{dy}{dx} = \frac{5}{y}$.
  - Find all values of A so that $y = 7e^{Ax}$ is a solution to $\frac{d^2y}{dx^2} = 9y$

- Differential Equations Verbally

  - *"Nature's voice is mathematics; its language is differential equations"*

  - Oil is being pumped continuously from a well at a rate proportional to the amount of oil left in the well. Let V be the amount of oil in the well and write a differential equation expressing the situation.

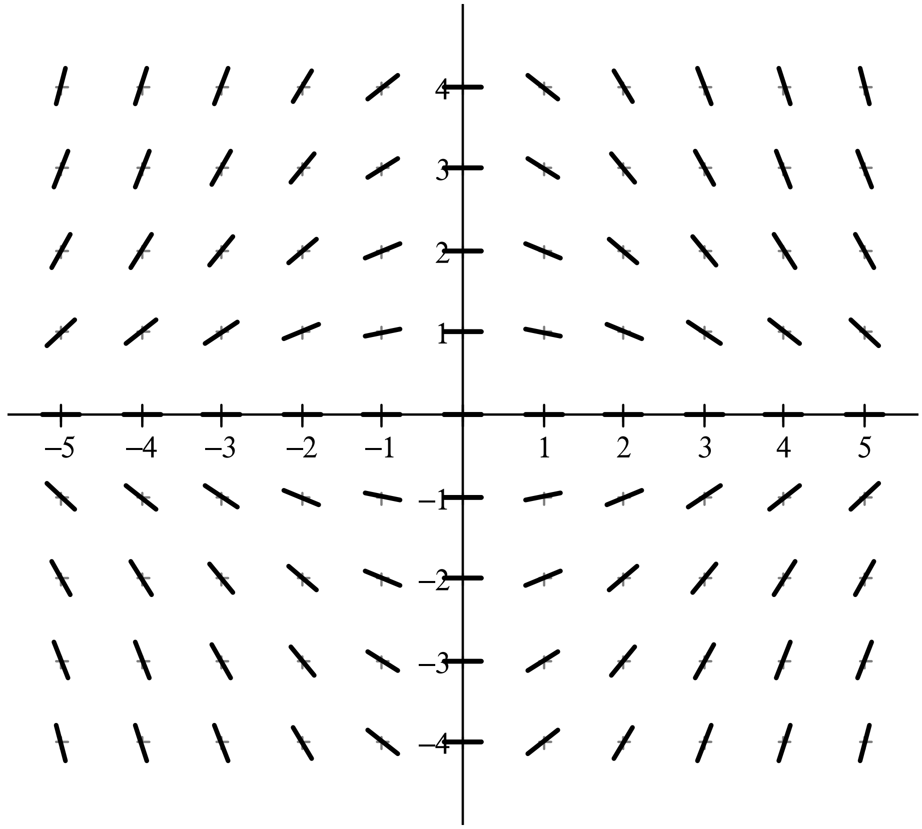
  - Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between the objects temperature and that of its' surrounding environment. Write a differential equation that models this situation

  - A rock has a constant downward acceleration of 9.8 meters per second per second. Write a differential equation that models this situation.

  - When a valve at the bottom of a water tower is opened the rate at which the level of the water in the tower changes is proportional to the square root of the level of the water in the tower. Express the rate of the water level in the form of a differential equation.

  - A flu epidemic spreads through Westlake High School, population 2500, at a rate proportional to the product of the umber of students already infected and the number of those not yet infected. Let $P(t)$ represent the number of students infected after t days. Express the statement above as a differential equation.

- Differential Equations Graphically



- Consider the possible solution for the given slope field. Since we do not know the differential equation describing the slope field we investigate this slope field graphically.

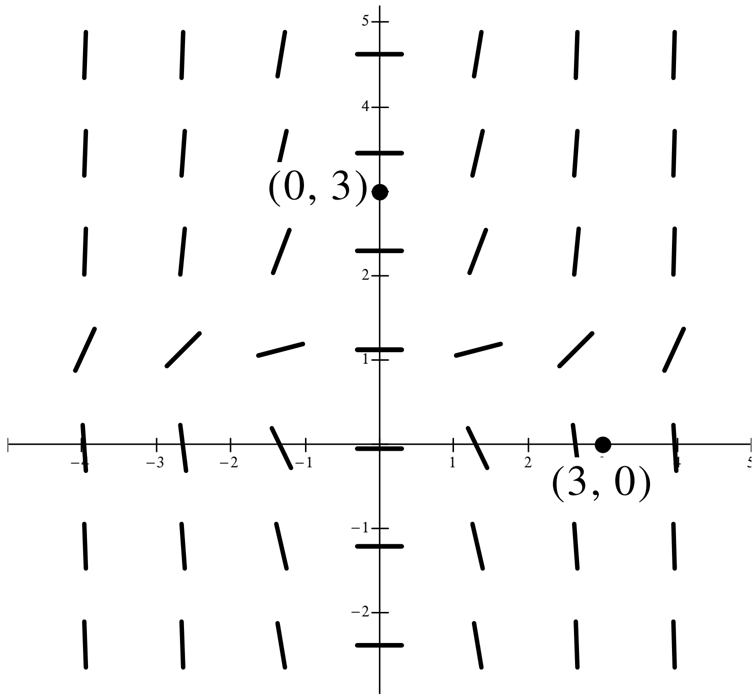
- Sketch the particular solution passing through (4, 1)

- Sketch the particular solution passing through (2, -1)

- Notice that all the slopes in Quadrant I are negative in fact the slope at the point (1, 1) is –0.2, try and write a differential equation that is plausible for this slope field.

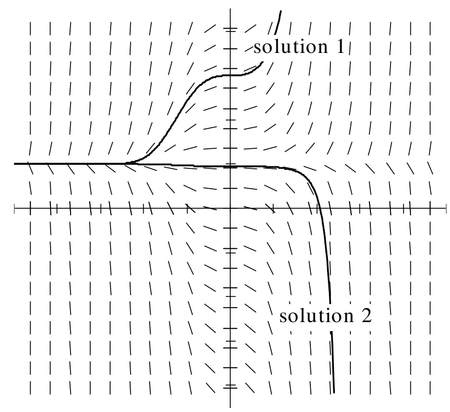
- Differential Equations Analytically

  - 2004 AB 6 "the first AB slope field"
  - Look at the slope field $\frac{dy}{dt} = x^2(y-1)$



  - Consider the solution, $y = f(x)$, of the differential equation with the initial value $f(0) = 3$. (solution 1)

  - Consider the solution, $y = f(x)$, of the differential equation with the initial value $f(3) = 0$. (solution 2)



  - Graphical solutions verses analytical solutions to the differential equation.

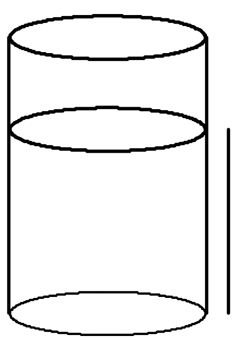
- Last comment on differential equations

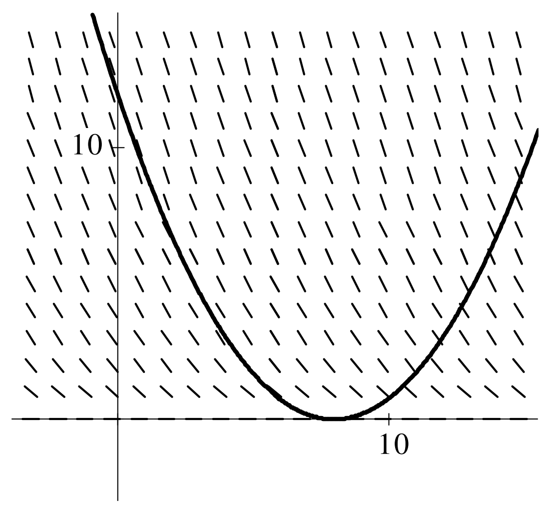
  - Suppose you fill a tall topless tin can with water, then punch a hole near the bottom with an ice pick. The water leaks quickly at first, then more slowly as the depth of the water decreases. In engineering or physics, you will learn that the rate at which water leaks out is directly proportional to the square root of its depth, *y*. Suppose at time 0 minutes, the depth y is 12 cm and $\frac{dy}{dt} = -3\,\text{cm / min}$. (From Paul Forester's <u>Calculus</u> Textbook)

    a) Write a differential equation stating that the instantaneous rate of change of *y* with respect to *t* is directly proportional to the square root of *y*. Find the proportionality constant.

    b) Plot the slope field for the differential equation above.

    c) Solve the differential equation analytically





  - Look at the 2003 AB 5 "the coffee pot problem" for similar inspiration.
  - Does the solution of a differential equation always fit the slope field?