Houston Area Calculus Teacher 1/30/2016

Experiments, Mishaps and Mistakes in AP Calculus.

- Card sort for f, f' and f". Ambiguity in understanding "decreasing at a increasing rate"

Developing of an understanding of Calculus helps many of my students do well on any AP Calculus assessments but when questioned deeply these same students seem only to have a surface understanding of the ideas of Calculus. Indeed even my own understanding of Calculus seems shallow as I repeatedly attempt to explain, both logically and intuitively, the ideas of Calculus; case in point "decreasing at a decreasing rate" and another "L'Hopital's Rule" and the list goes on and on. William Byers book <u>How Mathematicians Think</u> makes the case that focusing on the problematic and the ambiguous in mathematics often forms the contexts that provide productive creative insights. In my own teaching misunderstanding and mistakes are a catalyst to creative insights and stimulating activities about how to learn, understand and make sense of the ideas of Calculus.

- Differential Equations in Context

2012 AB/BC 5

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight.

At time t = 0, when the bird is first weighed, its weight is 20 grams. If B(t) is the weight of the bird, in grams, at time t days after

it is first weighed, then

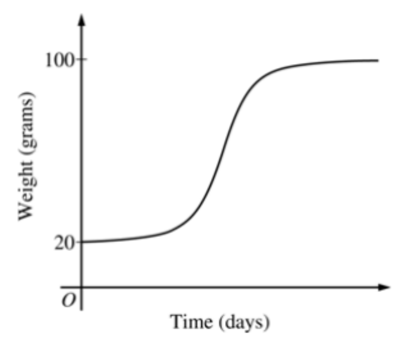
$$\frac{dB}{dt} = \frac{1}{5}(100 - B)$$

Let y = B(t) be the solution to the differential equation above with initial condition B(0) = 20.

a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.

c) Use separation of variables to find the particular solution to the differential equation with the initial condition $B(0) = 20$.



Possible alternate questions

b) Write the equation of the tangent line at t = 0.

d) Use the tangent line to estimate the weight of the baby bird at t = 2 days.

d) Is the estimate of the weight of the bird at t = 2 days an underestimate? Explain.

2011 AB/BC 5

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate the W will satisfy the differential equation $\frac{dW}{dt}=\frac{1}{25}(W-300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

a) Use the line tangent to the graph of W at t = 0 to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010.(time $t=\frac{1}{4}$)

b)Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t=\frac{1}{4}$.

c) Find the particular solution $W=W(t)$ to the differential equation $\frac{dW}{dt}=\frac{1}{25}(W-300)$ with initial condition $W(0)=1400$

- "Real" Understanding of the Contextual Differential Equations with experiments

Warm Up

Water is flowing into a large spherical tank at a constant rate. Let V(t) be the volume of the water in the tank at time t, and h(t) be the height of the water level at time t.

a) Give a physical interpretation of $\frac{dV}{dt}$ and $\frac{dh}{dt}$.
b) Are both $\frac{dV}{dt}$ and $\frac{dh}{dt}$ constant? Explain.
c) Is $\frac{dV}{dt}$ positive, negative or zero when the tank is one quarter full? Justify your answer.
d) Is $\frac{dV}{dt}$ increasing or decreasing when the tank is one quarter full? Explain.

- Torcelli's Law

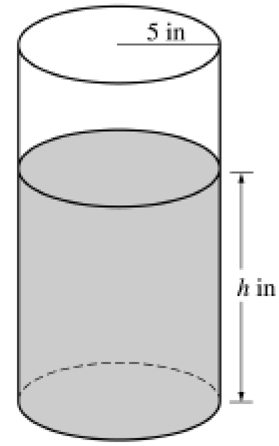
Consider the statement about water draining from a cylindrical tank taken from a current Calculus Textbook.

"The exit speed of the water flowing through the hole is proportional to the square root of the depth of the water."

Is it possible to use this statement and demonstrate a differential equation that models the physical situation to make meaning of what is going on?

2003 AB5

A coffeepot has the shape of cylinder with radius of 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second.



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

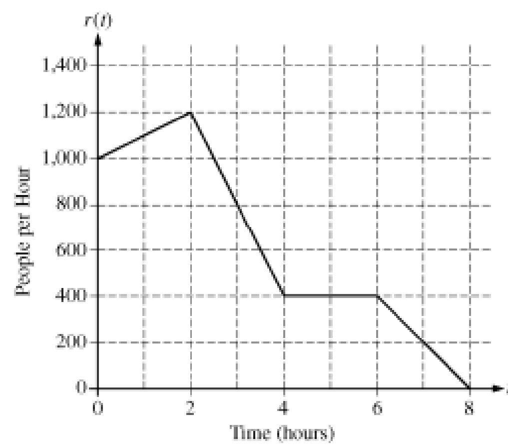
(b) Given that h = 17 at time t = 0, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time is the coffeepot empty?

- Touch and Feel to get a conceptual insight into AP Problems.

2010 AB#3

There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



(a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

(b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

(c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

(d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

2015 AB/BC #1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

2013 AB/BC #3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

- Counterexamples

Creating counterexamples requires an catalog of functions to pull from. Piecewise, absolute value, rational, $y = x^{2/3}$ and $y = \sin\left(\frac{1}{x}\right)$ are some that often come in handy.

Creating Counterexamples in Calculus some from *Counterexamples by Sergiy Klymchuk.

*A tangent to a curve at a point cannot touch the curve at infinitely many points.

*If f(x) is continuous and decreasing for all positive x and f(1) is positive, then f(x) has exactly one root.

*If g(a) = 0, then the rational function $R(x) = \frac{f(x)}{g(x)}$ (both f(x) and g(x) are polynomials) has a vertical asymptote at x = a.

If f(x) is continuous and increasing, then $\lim_{x\to\infty} f(x) = \infty$

*If a function is differential at a point then, it is differentiable at that point.

*If a function is twice-differentiable at a local minimum point, then its second derivative is positive at that point.

*If the function F(x) is the antiderivative of a function f(x), then $\int_a^b f(x) = F(b) - F(a)$.

If f(x) is continuous and differentiable and a tangent line at x = 1 is used to predict f(1.5) then the tangent line approximation cannot exactly predict f(1.5)

If f(x) is continuous on [a, b], then f'(x) exists for every point on (a , b)

If h(x) = f(x) g(x), then $h'(x) \neq f(x)g'(x)$