Workshop III Houston Area Calculus Teachers Scott Pass

Ideas for Integration: Recent focus on the direction of integration on the AP exam

Overview of Integrations questions from the 2018 and 2019 Exams

Q		2018 topic	Int	Der	2019 topic	Int	Der	
1	AB/BC	escalator	Y	Y	fish	Y	Y	
2	AB	motion	Y	Y	motion	Y	Y	
2	BC	plankton	Y	Y	polar	Y	Y	
3	AB/BC	graphical	Y	Y	graphical	Y	Y	
4	AB/BC	data	Y	Y	DE w/ application	Y	Y	
5	AB	function	N	Y	area and volume	Y	N	
5	BC	polar	Y	Y	improper integral	Y	Y	
6	AB	DE w/slope field	Y	Y	function with L'Hop	Y	Y	
6	BC	series	?	?	series	?	?	

The biggest trend I have noticed over the last 5 to 6 years is the imbedding of both derivative and integration topics within each individual question of the AP exam.

- Separable Differential Equations

Look at 2019 4 c and

- Choosing Integration in context

Look at 1 both years and BC 2 2018

- Integrating with techniques

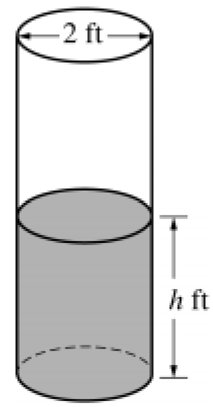
Look at 2019 3 a and b

- Interpreting with integration

Look at 2015 AB/BC 3

2019

4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



At time t = 0 seconds, the height of the water is 5 feet.

Use separation of variables to find an expression for h in terms of t.

- Ask carefully about solving differential equations

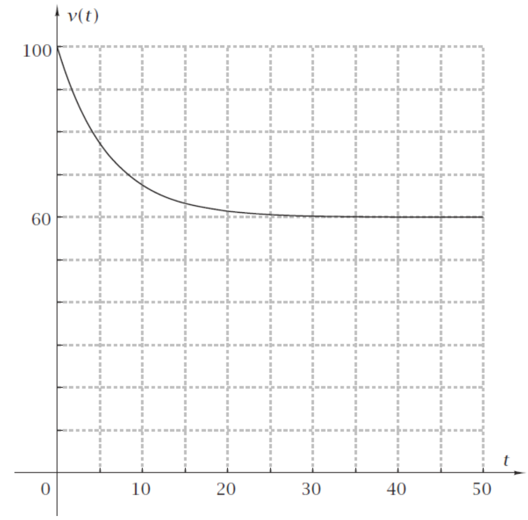
For # 1-3, Solve for P in terms of t, assuming a, b and k are nonzero constants

1. $\frac{dP}{dt} = k \cdot P$
2. $\frac{dP}{dt} = P - a$
3. $\frac{dP}{dt} - aP = b$
4. Given that $\frac{dA}{dr} = 2\pi r$ solve for A in terms of r

- Favorite idea: Areas and Volumes. Demonstrate cones volume.

Understanding the dimensional analysis in context is essential: touchy & feely approach to unit analysis.

As you drive on the highway you accelerate to 100 ft/s to pass a truck. After you have passed, you slow down to a more moderate 60 ft/s. The diagram shows the graph of your velocity, $v(t)$, as a function of the number of seconds, t, since you started slowing.

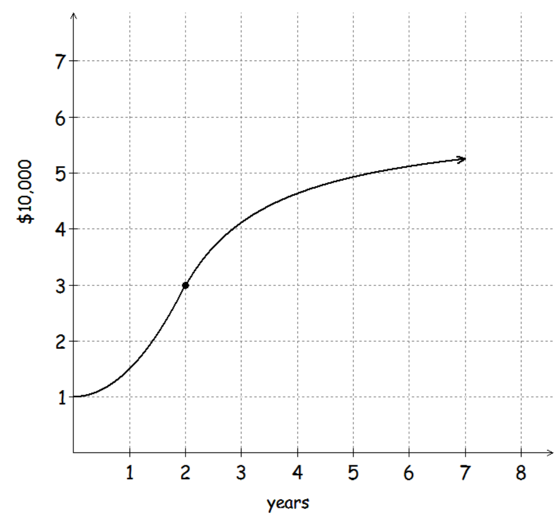


(a) On the interval from 20 to 50 seconds approximate your velocity?

(b) Estimate the distance travelled on the interval [20, 50]. Is your estimate an underestimate or an overestimate?

(c) Notice that v(t) has a variable velocity on the interval [0, 20]. Is there any way to determine the exact distance you travelled? Discuss

A stock portfolio increase in value over the first five years of the portfolios existence. The graph shows the value of the portfolio in ten of thousands of dollars as a function of the time in years. Estimate the rate of change of the value of the portfolio at t = 1, 2 & 3 years.



2018 BC 2

2. Researchers on a boat are investigating plankton cells in a sea. At a depth of h meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = 0.2h^2 e^{-0.0025h^2}$ for $0 \le h \le 30$ and is modeled by $f(h)$ for $h \ge 30$. The continuous function f is not explicitly given.

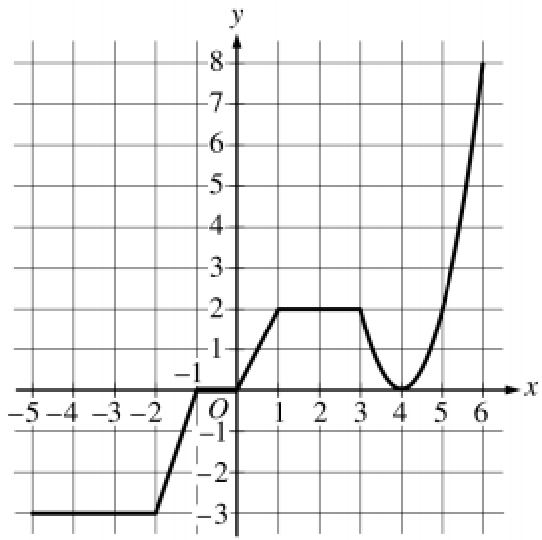
(a) Find $p'(25)$. Using correct units, interpret the meaning of $p'(25)$ in the context of the problem.

(b) Consider a vertical column of water in this sea with horizontal cross sections of constant area 3 square meters. To the nearest million, how many plankton cells are in this column of water between $h = 0$ and $h = 30$ meters?

(c) There is a function u such that $0 \le f(h) \le u(h)$ for all $h \ge 30$ and $\int_{30}^{\infty} u(h)\, dh = 105$. The column of water in part (b) is K meters deep, where $K > 30$. Write an expression involving one or more integrals that gives the number of plankton cells, in millions, in the entire column. Explain why the number of plankton cells in the column is less than or equal to 2000 million.

(d) The boat is moving on the surface of the sea. At time $t \ge 0$, the position of the boat is $(x(t), y(t))$, where $x'(t) = 662\sin(5t)$ and $y'(t) = 880\cos(6t)$. Time t is measured in hours, and $x(t)$ and $y(t)$ are measured in meters. Find the total distance traveled by the boat over the time interval $0 \le t \le 1$.

Integrating techniques must include symbolic representations.



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

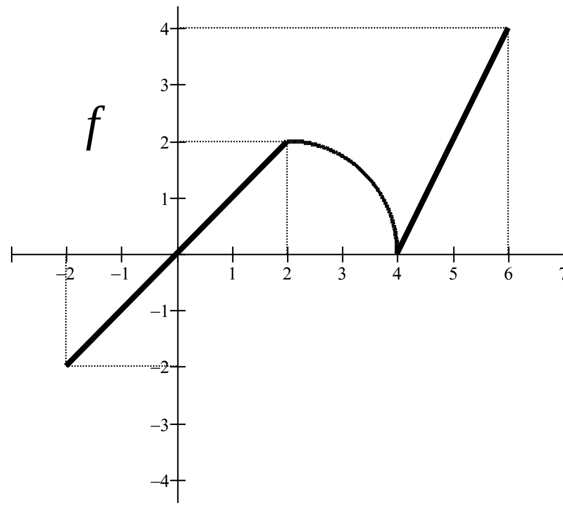
(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Given *f(x)* is defined on the closed interval x = [-2, 6]. The figure is composed of line segment and a quarter of a circle. It is known that the point $\left(3, \sqrt{3}\right)$ is on the graph of *f*.

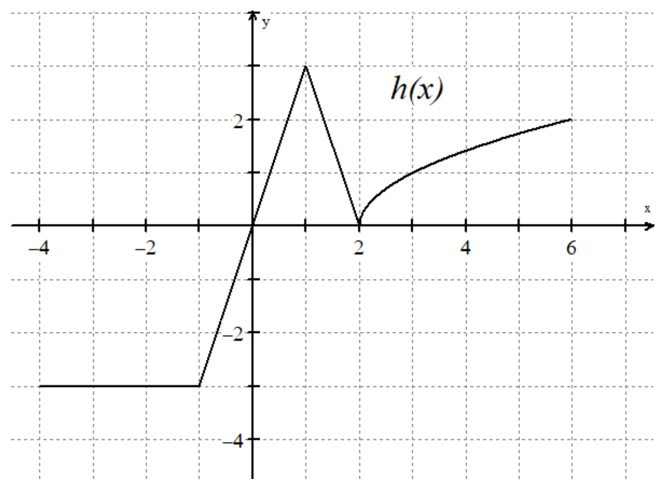


a) If $\int_{-4}^{4} f(x)dx = 10$, find the value of $\int_{-4}^{-2} f(x)dx$. Show your work!

b) Evaluate $\int_{3}^{6} 3(f'(x)+1)dx$

c) Evaluate $\int_{0}^{6} \left(f(x) + f'(x) \right) dx$

The graph of the continuous function h, the derivative of g, is shown. The function h is piecewise linear for $-4 \le x \le 2$, and $h(x) = \sqrt{x-2}$ for $2 \le x \le 6$



a) If g(1) = 2 find g(-4).

b) Evaluate $\int_{0}^{6} h(x)\, dx$

c) Given g(1) = 2, find an expression for g in terms of x for the interval $-1 \le x \le 1$.

d) Evaluate $\int_{1}^{3} h\left(2x\right) dx$

Interpreting reflects conceptual understanding.

You jump out of an airplane, before your parachute opens you fall faster and fast, but your acceleration decreases as you fall because of air resistance. The table gives your acceleration, a (in m/sec^2), after t seconds.

t	0	1	2	3	4	5
a	9.8	8.0	6.5	5.4	4.4	3.6

a) Evaluate a(5) – a(0). Interpret your answer in the context of the problem

b) Estimate and interpret $\int_0^5 a(t)dt$.

A cup of coffee is placed on the desk to cool. Let T(x) represent the temperature of the coffee at time x, where T is a differential function of x. The temperature of the coffee at selected times is given.

x (min)	0	4	7	12
T(x) (°F)	110	103	100	96

(a) Evaluate $\int_0^{12} T'(x)dx$. Explain the meaning of this definite integral.

(b) Explain the meaning of $\frac{1}{12}\int_0^{12} T(x)dx$. Approximate this expression.

If $W(t)$ represents the rate of earnings of a teacher born in 1965 measured in dollars/year and t is measured in years since 1965.

a) Interpret $\int_0^{20} W(t)dt = 0$. Is this reasonable?

b) Interpret $\int_0^{40} W(t)dt = 1000000$. Is this reasonable?

c) Interpret $\frac{1}{40-0}\int_0^{40} W(t)dt$

Calculus AB

Mixed Practice of Formulas

Consider the velocity function $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$ where v is measured in m/s. At t = 0, the position of the particle was 6 meters.

1. What is the velocity at t = 1?	2. How many times does the object change directions on the interval [0,4]?
3. What is the acceleration at t = 1?	4. What is the average velocity on the interval [0,4]?
5. What is displacement on the interval [0,4]?	6. What is the total distance traveled on the interval [0,4]?
7. What is the average acceleration on the interval [0,4]?	8. What is the change in velocity on the interval [0,4]?
9. What is the change in position from t = 0 to t = 4?	10. What is the position of the particle at t = 4?