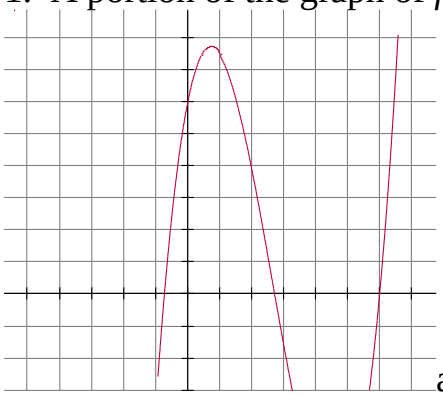
Approximating Function Values Using Secant and Tangent Lines

1. A portion of the graph of $f(x) = \frac{1}{2}x^3 - 4x^2 + 5x + 6$ is shown below.



a) Draw the secant line passing through the points (0,6) and (3, -3/2). Write the equation of the line passing through those points and use it to approximate the value of $f(2.2)$.

b) Draw the line tangent to the given graph at x=2. Write the equation for the tangent line and use it to approximate the value of $f(2.2)$.

c) Using your answers to parts b and c, determine an interval that includes the exact value of $f(2.2)$.

2. In the exercise above, the secant line underestimates the value of the function whereas the tangent line overestimates the value of the function. Illustrate a situation in which the tangent line would underestimate the value of the function and the secant line would overestimate the value of the function.

3. The table below gives selected values for a differentiable function $g(x)$.

x	-3	0	2	5	8
$g(x)$	2	-5	8	4	3

a) Name any interval or intervals in which there must exist a value c, $-3 \leq x \leq 8$, such that $g(c) = -3$. Justify your answer.

b) Write the equation of a line passing through the points where x = 2 and x = 5 and use the equation to approximate the value of $g(2.1)$.

c) Given $g'(2) = -1$, write the equation of the line tangent to $g(x)$ at x=2 and use it to approximate the value of $g(2.1)$.

4. The table below gives selected values of $h'(x)$ for a twice-differentiable function $h(x)$.

x	3	5	8	9	15	16
$h'(x)$	-4	-1	1	-2	7	-2

a) Is it possible for the graph of $h(x)$ to have exactly two critical points? Explain your reasoning.

b) Given $h(5) = 3$, write the equation of the line tangent to h(x) at x = 5 and use it to approximate the value of $h(5.2)$.

c) Given the information above and that $h(x)$ has a root at x=6, write the equation of a secant line between two known points and use it to approximate the value of $h(5.2)$.

d) Using your answers to parts b and c, determine an interval that includes the exact value of $h(5.2)$.

5. a) Given the function $j(x) = 2x^2 - 3x + 5$, write the equation of a line tangent to $j(x)$ at x = 1 and use it to approximate the value of $j(1.4)$.

b) Use your knowledge of the graph of $j(x)$ to determine if the answer in part a is an underestimate or overestimate of the actual value of $j(1.4)$. Explain your answer.

b) Write the equation of the secant line passing through x = 1 and x = 2 and use it to approximate the value of $j(1.4)$.