

Limits and Continuity

Why are limits so important in the study of Calculus?

1. Horizontal Asymptote: $y = c$ is a horizontal asymptote of $f(x)$ if and only if

2. Vertical Asymptote: $x = a$ is a vertical asymptote of $f(x)$ if and only if

3. Continuity: $f(x)$ is continuous at $x = a$ if and only if
 - i) $f(a)$ exists

 - ii)

 - iii)

4. Derivatives: $f'(x) =$

$$f'(a) =$$

Intermediate Value Theorem: If $f(x)$ is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$, then for every value m between $f(a)$ and $f(b)$ there exists at least one value c in (a, b) such that $f(c) = m$.

Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

- I. f has at least 2 zeros.
 - II. The graph of f has at least one horizontal tangent.
 - III. For some c , $2 < c < 5$, $f(c) = 3$.
- (A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II and III

Which of the following functions have horizontal asymptotes at $y = \frac{1}{5}$?

i) $f(x) = \frac{5x-1}{x+3}$

ii) $f(x) = \frac{x}{5x^2-3}$

iii) $f(x) = \frac{x^2}{5x^2-25}$

iv) $f(x) = \frac{4^x-6}{5 \cdot 4^x+1}$

v) $f(x) = \frac{1+3^x}{3^x+5}$

vi) $f(x) = \frac{\sin(x)}{5x+1}$

What are three situations on which limits fail to exist?

What are three types of discontinuities?

What are three situations in which derivatives fail to exist?

Illustrate using the definition of continuity a function that satisfies:

i, not ii



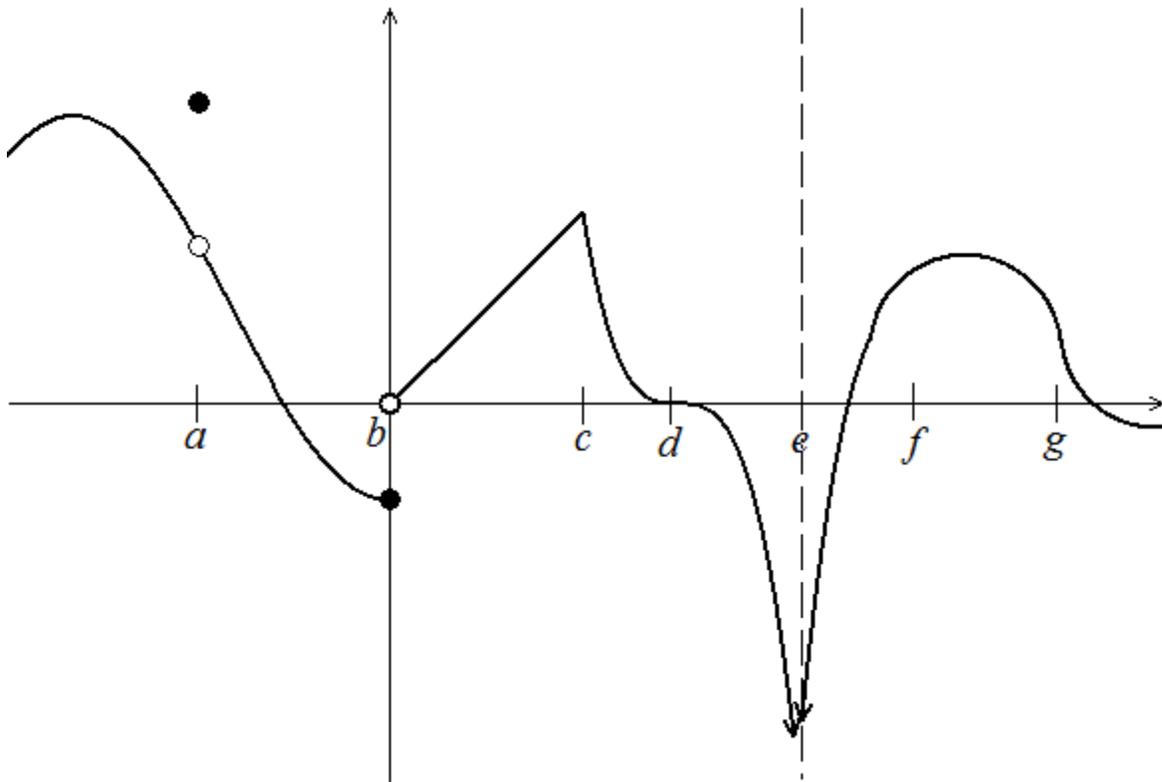
ii, not i



not i, not ii



i and ii, not iii



limit exists

limit does not exist

$f(x)$ is continuous

$f(x)$ is not continuous

$f(x)$ is differentiable

$f(x)$ is not differentiable

Show a function that is continuous, but not differentiable

Can a function be differentiable, but not continuous?

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & x < 1 \\ \frac{2x^2 - 6x}{x - 3} & x > 1, x \neq 3 \\ 2 & x = 1, 3 \end{cases}$$

Is $f(x)$ continuous at $x = 1$? Justify your answer using the definition of continuity.

Is $f(x)$ continuous at $x = 3$? Justify your answer using the definition of continuity.

Is $f(x)$ differentiable at $x = 1$? Why or why not?