Rivers and streams flow into Lake Buchanan and heavy rains can cause flooding. The Lower Colorado River Authority, LCRA, starts monitoring the level of water in the lake when the heavy rains start and will open floodgates on the Lake Buchanan dam to allow water to flow downstream to minimize flooding damage.

Let $E(t) = t + \sin t$ be the rate of water entering Lake Buchanan at a **hundred cubic feet/hour** due to the heavy rains.

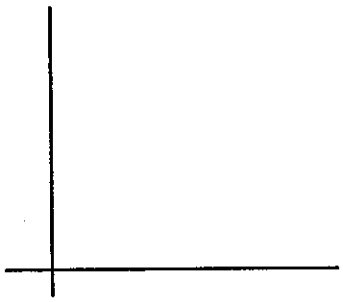
Let $R(t) = \begin{cases} 2, & 1 \le t < 6 \\ 10.5, & 6 < t \le 12 \end{cases}$ be the rate of water being released from the floodgates at a **hundred cubic feet/hour**.

Notice the floodgates are closed on [0, 1) so $R(t) = 0$ *on* $[0,1)$

Answer the following showing <u>all</u> set-ups and express answers correct to three decimal places with <u>correct units</u>. Time t represents hours and $0 \le t \le 12$.

a) Graph $E(t)$ and $R(t)$ in an appropriate viewing window labeling axes with units.

b) At what times t is the rate of water entering the lake equal to the rate of the water being released?



c) *Since* $E(1) = 1.841$ *and* $E(4) = 3.243$, explain why there must be a value t for $1 \le t \le 4$ such that $E(t) = a$, *where* $1.841 < a < 3.243$. Describe what this means in relation to the given situation.

d) **[non-calculator]**
What is the average rate of change of $E(t)$ on $[0, \pi]$? Explain why there must be a value of t for which the average rate of change of $E(t)$ is equal to the instantaneous rate of change of $E(t)$ on $[0, \pi]$ and find that exact value of t.

e) How much water entered the lake in the first hour of monitoring?

f) Find the average value of $R(t)$ *on* $[1,12]$.

g) From $t = 2$ *to* $t = 12$ hours, give the time intervals when more water was entering the lake than was being released. Determine how much more water entered than was released in those time intervals.

h) If A is the amount of cubic feet of water in Lake Buchanan at time $t = 0$ represent the amount of water, Q, in the lake at $t = 7$.

i) At what time t will the amount of water in the lake return to the amount at time $t = 0$?

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
-2	4	3	2	8
3	5	-4	-2	7
5	1	-1	3	6

1. Find f'(3) for each of the following:
   a) $f(x) = g(x) / h(x)$

   b) $f(x) = h(x) - g(x)$

   c) $f(x) = h(g(x))$

   d) $f(x) = [h(x)]^4$

   e) $f(x) = g(x)\, h(x)$

   f) $f(x) = h(x) / g(x)$

   g) $f(x) = g(h(x))$

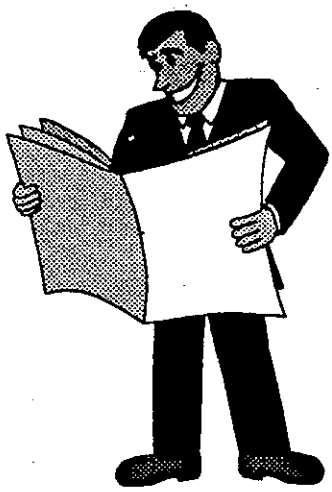
   h) $f(x) = [g(x)]^3$

2. Find $d/dx\, [g^{-1}(x)]$ at $x = 5$.

3. Let $f(x) = g(h(x))$
   a. Is there a point c, $3 \leq c \leq 5$, such that $f(c) = 9/2$? Explain.

   b. Is there a point c, $3 \leq c \leq 5$, such that $f'(c) = \frac{1}{2}$

   c. Is there a point c, $3 \leq c \leq 5$, such that $f''(c) = -45/2$



DERIVATIVES IN THE NEWS

Read the following excerpts from newspaper articles, looking for statements that deal with the concepts of the first and second derivative. Answer the questions following each excerpt. Then look for an article such as this in newspapers and magazines. When you find one, identify the parts of the article that deal with a first and/or second derivative and translate those parts into an appropriate mathematical statement about some function.

Excerpt #1: (from a 1993 newspaper article)
Sales of new homes in September unexpectedly soared to nearly a seven-year high and the government's economic forecasting gauge now points to "a sustainable recovery with constant growth," federal officials and analysts said Tuesday.

If E represents the level of the U. S. economy as a function of time, then analysts are predicting
(a) E' will be positive/negative/zero/can't tell?
(b) E'' will be positive/negative/zero/can't tell?

Excerpt #2: (from a 1993 newspaper article)
Police have tallied 376 homicides in the city since the beginning of the year – up from 365 for the same period in 1992 – many of them related to gangs and a drug trade that runs rampant in some neighborhoods.

If H represents the number of homicides in this city as a function of time, then
(a) H is positive/negative/zero/can't tell?
(b) H' is positive/negative/zero/can't tell?
(c) H'' is positive/negative/zero/can't tell?

Excerpt #3: (from a 1993 newspaper article)
IBM Corp. is starting to pull out of the red, company officials said Tuesday. The computer maker took a loss of 12 cents a share in the third quarter. That compares with a per-share loss of $4.87 a year ago, ...

If P represents IBM's net profit per share, then
(a) P is positive/negative/zero/can't tell?
(b) P' is positive/negative/zero/can't tell?
(c) P'' is positive/negative/zero/can't tell?

1

X	Y1
0	-1
5	9
10	19
15	29
20	39
25	49
30	59

X=0

2

X	Y1
0	-3
1	-2
2	0
3	4
4	12
5	28
6	60

X=0

3

X	Y1
0	-2
1	-2.5
2	-2.75
3	-2.875
4	-2.938
5	-2.969
6	-2.984

X=0

4

X	Y1
0	.69315
1	1.0986
2	1.3863
3	1.6094
4	1.7918
5	1.9459
6	2.0794

X=0

5

X	Y1
0	1
2	0
4	-1
6	-2
8	-3
10	-4
12	-5

X=0

6

X	Y1
-70	4.9628
-60	4.8122
-50	4.6347
-40	4.4188
-30	4.1431
-20	3.7612
-10	3.1355

X= -70

(z)

$$f'(x) > 0$$

$$f''(x) > 0$$

(v)

$$f'(x) > 0$$

$$f''(x) < 0$$

(y)

$$f'(x) < 0$$

$$f''(x) > 0$$

(x)

$$f'(x) < 0$$

$$f''(x) < 0$$

(u)

$$f'(x) > 0$$

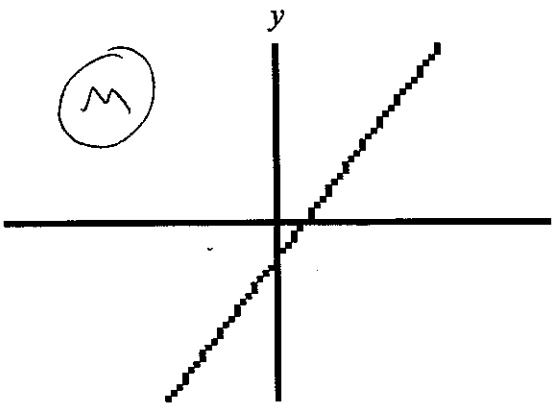
$$f''(x) = 0$$

(w)

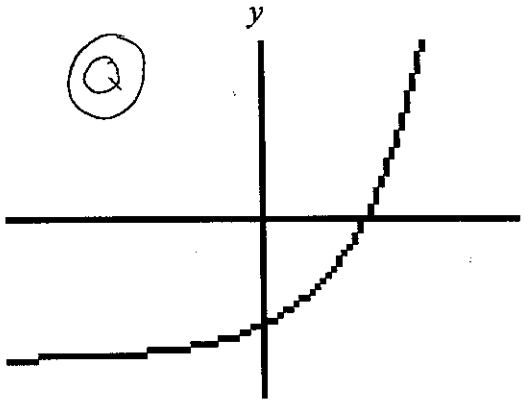
$$f'(x) < 0$$

$$f''(x) = 0$$

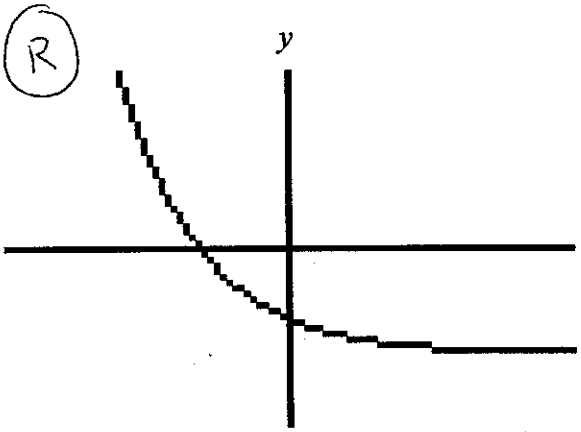
M



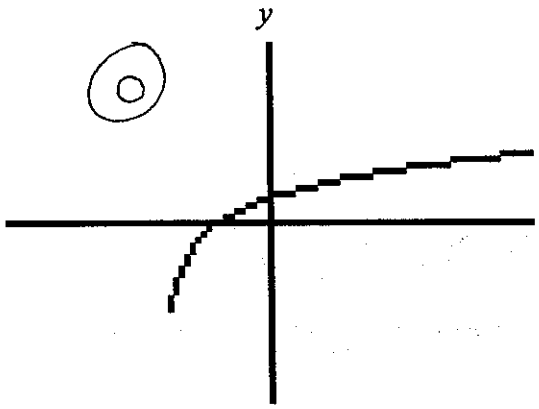
Q



R



O



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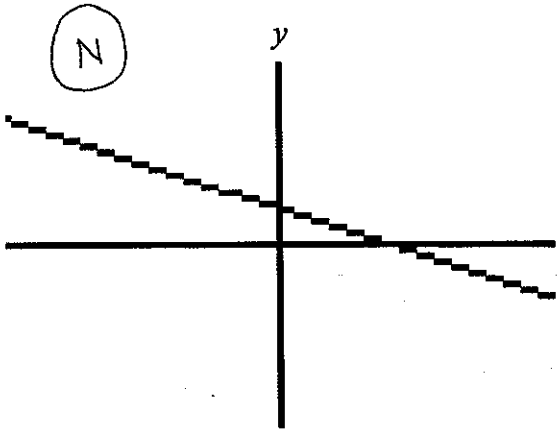
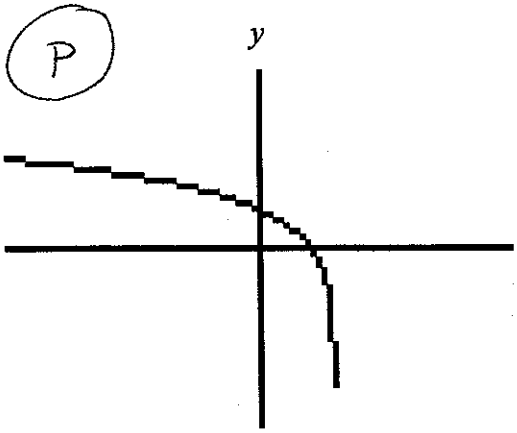


P



(A)

Increasing at an increasing rate

(B)

Decreasing at an increasing rate

(F)

Decreasing at a decreasing rate

(D)

Increasing at a decreasing rate

(E)

Increasing at a constant rate

(C)

Decreasing at a constant rate