My history of piece-wise functions on the AP Exam

	Form A	Form B
2002	#6d	
2003	#6	
2004		
2005	#5	
2006		
2007	#2	
2008		#5
2009	#6	#3
2010	#1	
2011	#4d, #6	#2, #3

Dealing with piece-wise functions on the AP exam

- What does it mean for a function to be continuous at a point $x = c$?

- What does it mean for a function to be differentiable at a point $x = c$?

- If a function is continuous, does that guarantee that it is differentiable?

- If a function is differentiable, does that guarantee that it is continuous?

- Is the function $f(x) = \begin{cases} e^{4x}, & x < 0 \\ x^2 + 4x, & x \leq 0 \end{cases}$ differentiable?

- Example 1: $f(x)=\begin{cases} 3x^2-2; & 0\le x\le 2 \\ 2x+6; & 2<x\le 6 \end{cases}$

a) Is $f(x)$ continuous on at x = 2?

b) Is $f(x)$ differentiable at x = 2?

c) Find the average rate of change of $f(x)$ on the interval $0 \le x \le 6$. There is no point c, $0 < c < 6$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

d) Find the value of $\int_0^6 f(x)\,dx$.

- Example 2: $g(x) = \begin{cases} ax+b; & -3 \le x < 1 \\ cx^3 - 3; & 1 \le x \le 4 \\ -1; & x = 1 \end{cases}$

a) Find the values of *a*, *b,* and *c* that will make $g(x)$ continuous and differentiable on the interval [-3, 4].

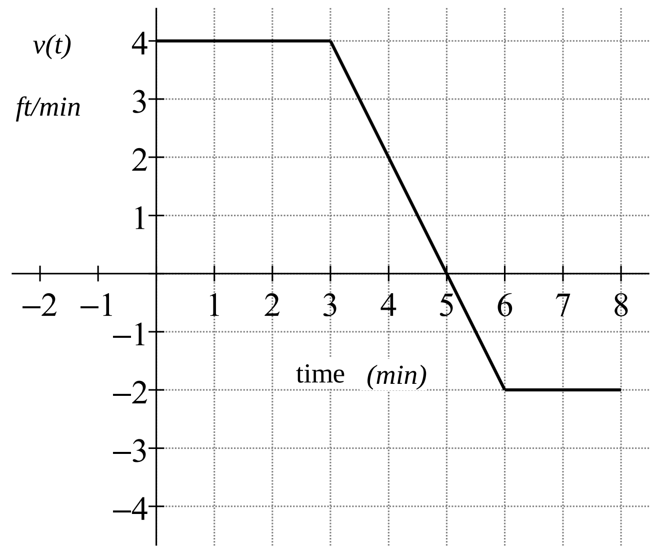
b) Find the average rate of change of $g(x)$ on the interval [-3, 4].

c) Does the Mean Value Theorem guarantee a point $x = c$ in the interval [-3, 4] such that $g'(c)$ = the value found in part b? If so, find the value of c.

d) Write, but do not evaluate, an integral expression for the average value of $g(x)$ on the interval [-3, 4].

- Example 3:

The graph below gives the velocity of a particle moving along the x-axis at time *t*.



a) Write a piece-wise function to represent the velocity of the particle for $0 \le t \le 8$.

b) For each of v'(3) and v'(4), find the value or explain why it does not exist. If a value does exist, explain its meaning in the context of the problem. Indicate units of measure.

c) Write a piece-wise function to represent the acceleration of the particle for $0 < t < 8$..

d) Find the total distance traveled by the particle for $0 \le t \le 8$.

e) Is the speed of the particle increasing or decreasing at $t = 6$? Justify your answer.

f) If the particle is initially located at $x = 3$, find its position at $t = 8$.

- Example 4: A tank contains 60 gallons of molasses. At $t = 0$, molasses begins flowing into the tank at a rate modeled by $r(t)$, measured in gallons per hour, where r is given by the piecewise defined function

$$r(t) = \begin{cases} 4e^{\frac{x}{3}-2} & \text{for } 0 \le t \le 6 \\ \frac{1}{2}x^2 - 2x - 2 & \text{for t} > 6 \end{cases}$$

a) Is r continuous at $t = 6$? Show the work that leads to your answer.

b) Find $r'(8)$. Using correct units, explain the meaning of that value in the context of this problem.

c) Using correct units, explain the meaning of $\frac{\int_2^6 4e^{\frac{x}{3}-2}dx + \int_6^8 \frac{1}{2}x^2 - 2x - 2\,dx}{8-2}$ in the context of the problem.

d) Write, but do not solve, an equation involving an integral to find the time S when the amount of molasses in the tank is 100 gallons.

- Example 5: Let *f* be a function defined by $f(x) = \begin{cases} 2\sqrt{x} & \text{for } 0 \leq x \leq 4 \\ -2x + 12 & \text{for } 4 \leq x \leq 6 \end{cases}$

a) Show that *f* is continuous at $x = 4$.

b) For $x \neq 4$, express $f'(x)$ as a piecewise-defined function. Find the value of *x* for which $f'(x) = 3$.

c) Find the average rate of change of *f* on the interval [1,5]. Does the Mean Value Theorem applied on this interval guarantee a value of c, $1 < c < 5$, such that $f'(c)$ is equal to this average rate of change? Why or why not?

d) Find the average value of *f* on the interval [1,5] and all values where *f* equals that average.

e) Let R be the region enclosed by $f(x)$ and the x-axis. Find the area of region R.

f) Write, but do not evaluate, an integral expression for the volume of the figured formed by rotating region R around the line $y = -1$.

g) The region R is the base of a solid. For each *y*, where $0 \leq y \leq 4$, the cross section of the solid taken perpendicular to the *y*-axis is a rectangle whose base lies in R and whose height is $3y$. Write, but do not evaluate, an integral expression that gives the volume of this solid.