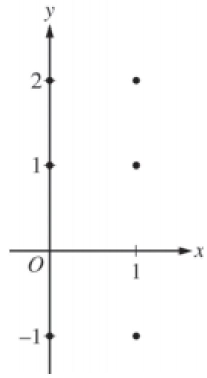
Expecting the Unexpected on the AP Exam
HACT 2015

2015 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

4. Consider the differential equation $\frac{dy}{dx} = 2x - y.$

   (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

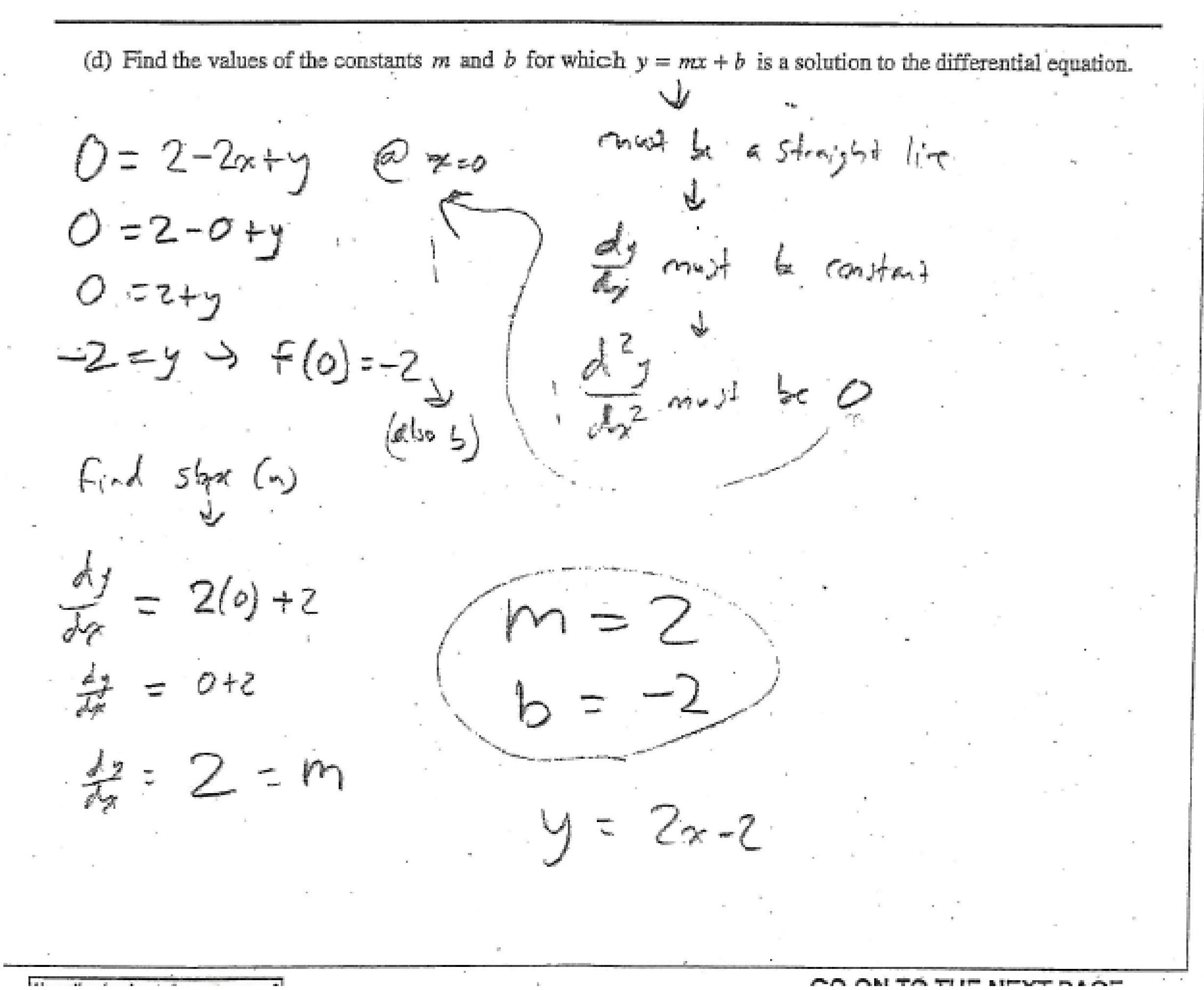


   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

   (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

   (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Here's a student's super clever solution:



(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

$0 = 2 - 2x + y$ @ $x=0$

$0 = 2 - 0 + y$

$0 = 2 + y$

$-2 = y \rightarrow f(0) = -2$ (also b)

Find slope (m)

$\frac{dy}{dx} = 2(0) + 2$

$\frac{dy}{dx} = 0 + 2$

$\frac{dy}{dx} = 2 = m$

must be a straight line

$\frac{dy}{dx}$ must be constant

$\frac{d^2y}{dx^2}$ must be 0

$m = 2$

$b = -2$

$y = 2x - 2$

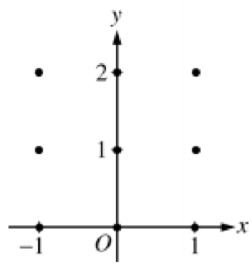
And here's another super clever solution given to me by a teacher:

So, did that problem just come out of nowhere?

2007 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
   (Note: Use the axes provided in the exam booklet.)



   (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

   (c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

   (d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

   (a) Find the time t at which the particle is farthest to the left. Justify your answer.

   (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

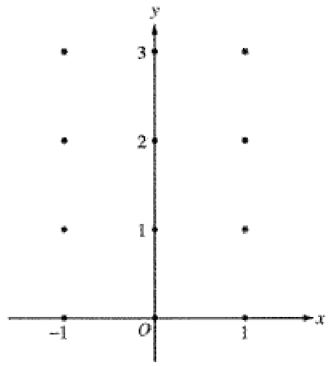
Add the word differential right before equation in part b!

The question below lead to one of my favorite wrong answers, but also reminds us of the necessity of working with both the differential equation and the slopefield.

(No Calculator)

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

This next question had one of my favorite right answers and taught me something about differential equations that I hadn't thought of before.

1997 AB6/BC6

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

Here are some other interesting differential equation problems I found while I was rummaging around:

2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

5. Consider the differential equation $\frac{dy}{dx} = 5x^2 - \frac{6}{y-2}$ for $y \neq 2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = -4$.

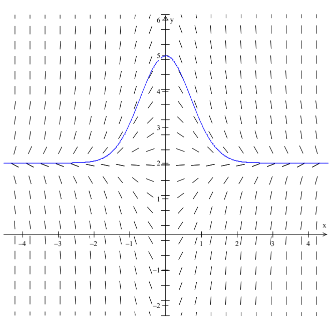
(a) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(-1, -4)$.

(b) Is it possible for the x-axis to be tangent to the graph of f at some point? Explain why or why not.

From the practice exam posted at AP Central behind the "audit" wall:

5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x\to\infty} f(x) = 2$.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.



So, let's do something interesting with another differential equation from 2015:

2015 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

   (a) Write an equation for the line tangent to the curve at the point $(-1, 1)$.

   (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

   (c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.