CALCULUS IMPROV

On Oct 11, 2014, I spoke before a group of Houston Area Calculus Teachers on the topic of writing your own AP-style questions. In my mind, an AP-style question is not necessarily multiple choice or free response in format, but are questions that provoke students to consider the material in new ways and require thinking at the upper levels of Bloom's taxonomy. I brought along several examples I had developed in advance (available in a separate handout). I also wanted to demonstrate to the teachers in attendance that these types of questions could be written in fairly short amounts of time so I decided to play a game I have dubbed "CALCULUS IMPROV."

In traditional comedy improv, actors might be given a character, a setting and a prop and have just a few minutes to come up with a sketch. In my version of the game, I asked the teachers to make choices from three different categories (next page) and I would develop an AP-style question in under five minutes. The ten questions I developed that day are included in this handout. For some questions I began with an idea at the workshop and have since fleshed them out further. These questions are not intended to be perfect and can definitely be improved upon. Feel free to make use of them in any way you choose.

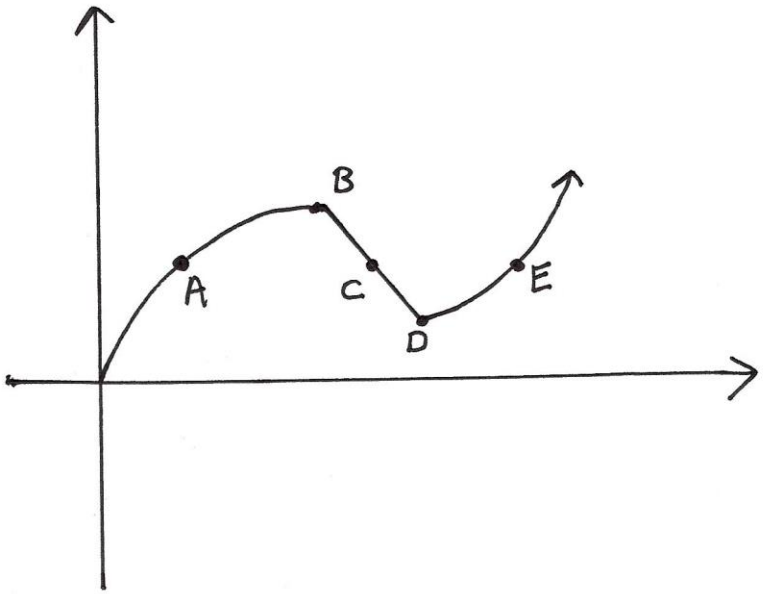
I hope you will make use of these questions somehow in your AP Calculus classes, but more importantly, will be inspired to play this game yourself and become more adept at developing your own questions.

Categories

Make one choice from each of the categories below and I will try to write an AP-style question in less than five minutes.

		Theorems
Numerical	Area/Volume	Tangent lines
Graphical	Differential Equations	Max/min
		Inflections points, concavity
Verbal	*f, f', f''* relationships	Derivatives
		Integrals
Analytic	Contextual Questions	Piece-wise
	Motion	Limits and Continuity
		Riemann Sum
		Asymptotes

Categories: Graphical, *f, f', f''* relationships, piece-wise



At which of the points above would the following be true?

i) $f'(x) > 0, f''(x) < 0$

ii) $f'(x) < 0, f''(x) = 0$

iii) $f'(x) > 0, f''(x) > 0$

Categories: Numerical, *f, f', f''* relationships, piece-wise

A student has a chart with information about the derivative values at various points.

x	-2	0	1	4
$f'(x)$	Positive	0	Negative	Negative
$f"(x)$	Negative	Negative	Does not exist	0

Which of the following piece-wise functions meet these conditions:

i) $f(x) = \begin{cases} x^3 & x < 1 \\ x & x \geq 1 \end{cases}$

ii) $f(x) = \begin{cases} 1 - x^2 & x < 1 \\ -2x + 2 & x \geq 1 \end{cases}$

iii) $f(x) = \begin{cases} x & x < 1 \\ 2 - x^2 & x \geq 1 \end{cases}$

Follow up questions: Make charts similar to the one above for the other two piece-wise defined functions.

x	-2	0	1	4
$f'(x)$				
$f"(x)$				

x	-2	0	1	4
$f'(x)$				
$f"(x)$				

Choose one of the functions above and use the definition of continuity to prove it is continuous for $x = 1$.

Categories: Analytic, Motion, Theorems

The position of a particle is given by $x(t) = 2t^2 + 5t - 12$ for time values $-2 \leq t \leq 2$.

Is there guaranteed to be a time in the given interval when the particle is at the origin? How could you prove it?

Is there guaranteed to be a time in the given interval when the velocity $v(t) = 5$? How could you prove it?

Categories: Analytic, Contextual Questions, Riemann Sum

The rate at which water leaks from a barrel is given by the function $R(t) = \sqrt{t+2}$, measured in gallons per minutes. Using a right Riemann sum with three subintervals and t values of t = 0, t = 2, t = 7 and t = 14, approximate the volume of water that has leaked out during the time interval $0 \leq t \leq 14$ minutes.

Categories: Verbal, *f, f', f''* relationships, inflection points and concavity

Which of the following statements indicates a point of inflection?

i) f changes from increasing to decreasing

ii) f goes from increasing at a decreasing rate to increasing at an increasing rate

iii) f' changes from positive to negative

iv) f' changes from increasing to decreasing

v) tangent lines go from above the curve to below the curve

vi) f'' changes from above the x-axis to below the x-axis

Categories: Analytic, *f, f', f''* relationships, max/min

Given, $f'(x) = (x-3)(x+2)^2$ and using the second derivative test, what can you conclude about $x = 3,\ x = -2,$ and $x = 5$?

Categories: Numerical, differential equations, derivatives

x	-2	1	5	7
y	4	-3	4	1

Given $\frac{dy}{dx} = 3x^2 + 4y$, what can you conclude about the concavity of the graph at the point where *x* =1? Does this point represent a relative maximum or minimum? Explain your answer.

Categories: Verbal, *f, f', f''* relationships, piece-wise

For what domain values is the function given below increasing at a decreasing rate?

$$f(x) = \begin{cases} x^2 & x \le 0 \\ 3x+1 & 0 < x < 3 \\ \sqrt{x-3} & x \ge 3 \end{cases}$$

Categories: Verbal, Area/volume, limits and continuity

Find the area of the region enclosed between the function $f(x) = e^{-2x}$ and the x-axis from x = 0 to x = k. Find the limit of the area as $k \to \infty$.

Hey! This last problem is kind of lame, but if you have something better, send it to me! dixross@austin.rr.com