Houston Area Calculus Teachers
Oct 11, 2014

Presenter: Dixie Ross

These are the problems that participants worked while I did my "calculus improv" as explained in my other handout.

The first two pages are inspired by the 2014 AP exam question #5. We have previously seen problems where students had to use a chart to determine derivative values. The new twist here is the use of the Fundamental Theorem of Calculus! I love the AP exam because it's always showing me a new way to think about the material that I teach.

Use the chart below to evaluate each of the following derivatives.

x	2	3	5	7	8
$f(x)$	0	7	e	-1	2
$g(x)$	4	2	3	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$f'(x)$	1	5	6	8	-4
$g'(x)$	-2	10	-3	9	1

1. $\frac{d}{dx}[f(x)g(x)]\Big|_{x=2}$

2. $\frac{d}{dx}[g(f(x))]\Big|_{x=3}$

3. $\frac{d}{dx}[e^{f(x)}]\Big|_{x=2}$

4. $\frac{d}{dx}[\ln(f(x))]\Big|_{x=5}$

5. $\frac{d}{dx}[\sin g(x)]\Big|_{x=7}$

6. $\frac{d}{dx}\left[(g(x))^3\right]\Big|_{x=2}$

7. $\frac{d}{dx}[f(g(x))]\Big|_{x=5}$

8. $\frac{d}{dx}[f^{-1}(x)]\Big|_{x=7}$

9. $\frac{d}{dx}[g(\ln x^2)]\Big|_{x=e}$

10. $\frac{d}{dx}\left[\frac{f(x)}{(g(x))^2}\right]\Big|_{x=3}$

Use the chart to find the value of each of the following definite integrals.

x	2	3	5	7	8
$f(x)$	0	7	e	-1	2
$g(x)$	4	2	3	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$f'(x)$	1	5	6	8	-4
$g'(x)$	-2	10	-3	9	1

1. $\int_2^7 f'(x) - g''(x)dx$

2. $\int_3^5 \frac{f'(x)}{f(x)} dx$

3. $\int_2^5 f'(x)[f(x)]^2 \, dx$

4. $\int_7^8 g'(x) \sin\big(g(x)\big) \, dx$

5. $\int_3^8 f'(x) g'\big(f(x)\big) \, dx$

6. $\int_5^5 \frac{f(x)}{g(x)} dx$

7. $\int_2^7 f'(x) e^{f(x)} \, dx$

8. $\int_0^{2\pi} \sin x \, g'(2\cos x) \, dx$

9. $\int_3^8 \frac{g'(x)}{\big(g(x)\big)^2} dx$

10. $\int_2^8 f'(x)\sqrt{f(x)} \, dx$

11. $\int_3^7 f'(x)g(x) + g'(x)f(x) \, dx$

Answers

Derivatives:

1. 4 2. 45 3. 1 4. 6/e 5. $\frac{9\sqrt{3}}{2}$

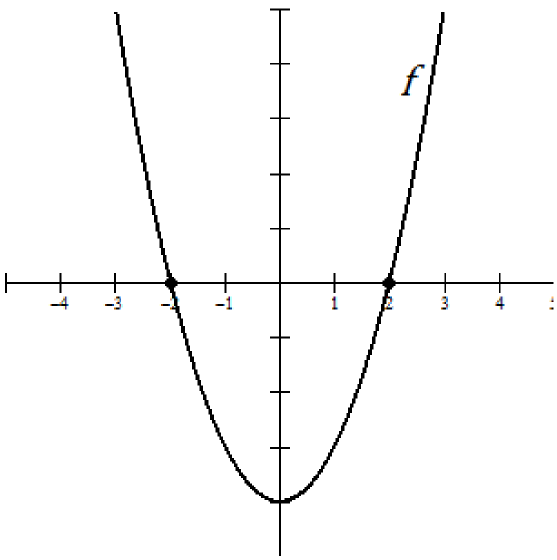
6. -96 7. -15 8. 1/5 9. -4/e 10. -65/4

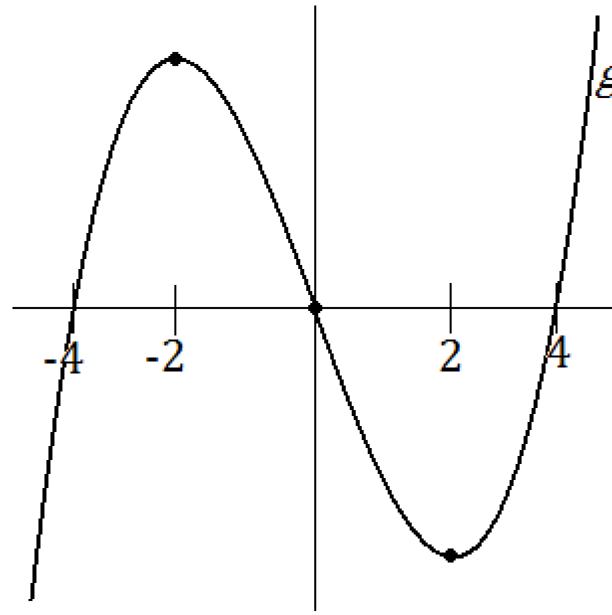
Integrals:

1. -12 2. 1 – ln 7 3. $\frac{e^3}{3}$ 4. $\frac{\sqrt{3}-1}{2}$ 5. $4 - \pi/6$

6. 0 7. 1/e – 1 8. 0 9. -3/π – ½ 10. $\frac{4\sqrt{2}}{3}$

11. –π/6 -14





The graphs of two differentiable functions *f* and *g* are shown above. Choose a conclusion from the list below for each of the following functions at the indicated value.

i) $h'(a) < 0$
ii) $h'(a) = 0$
iii) $h'(a) > 0$
iv) $h'(a)$ is undefined
v) There is not enough information to determine anything about $h(a)$.

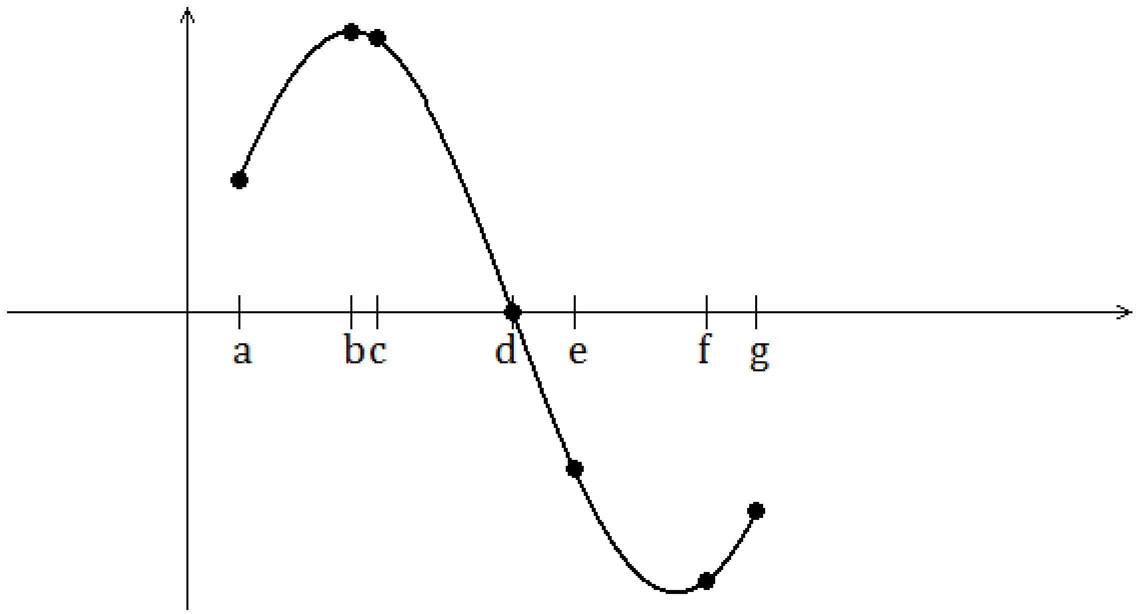
1. $h(x) = f(x) - g(x)$ $\quad h'(0)$

2. $h(x) = f(x)g(x)$ $\quad h'(2)$

3. $h(x) = \frac{f(x)}{g(x)}$ $\quad h'(0)$

4. $h(x) = f\big(g(x)\big)$ $\quad h'(0)$

5. $h(x) = \big(f(x)\big)^3$ $\quad h'(-1)$



What is the *x*-coordinate of the point given above that meets the following conditions?

i) $f'(x) = 0$

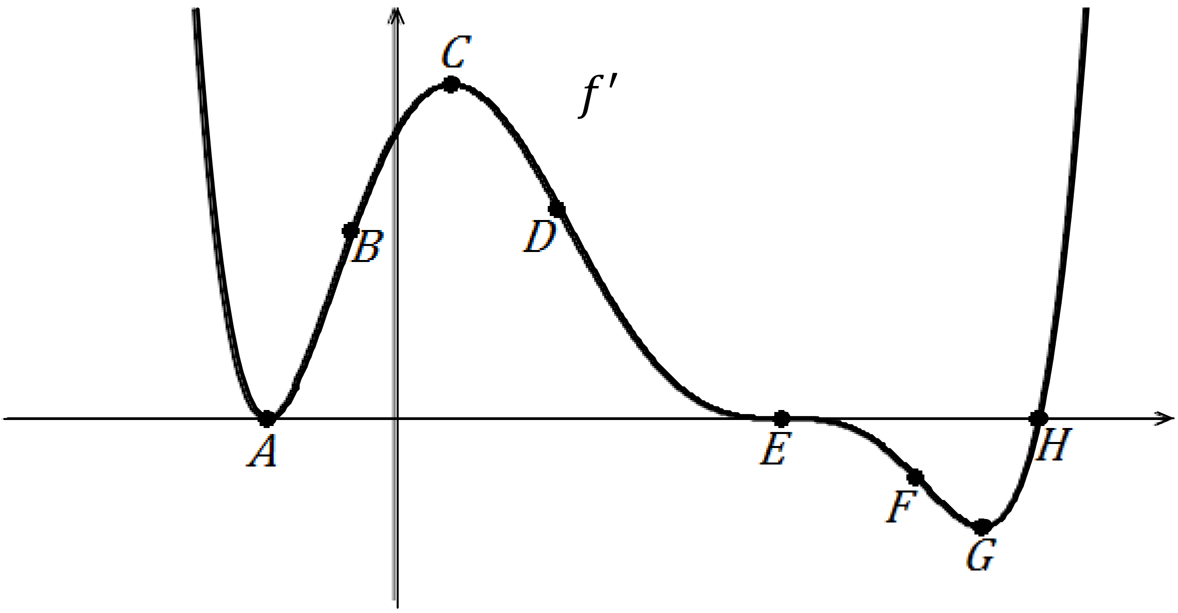
ii) $f''(x) = 0$

iii) $f''(x) < 0$

iv) $f'(x) < 0, f''(x) > 0$

v) line tangent to the graph of $f(x)$ is parallel to secant line through $\left(a, f(a)\right)$ and $\left(d, f(d)\right)$

vi) tangent line is horizontal



Graph of f'

Based on the graph of f' given, identify the following points on (x) .

i) critical points

ii) relative maximum

iii) relative minimum

iv) points of inflection

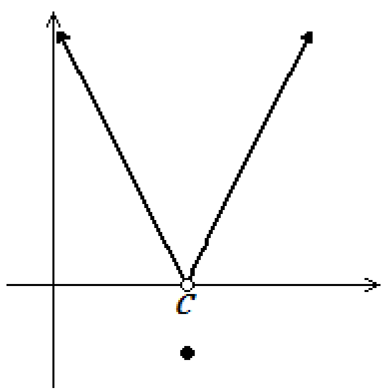
Which does not belong?

$f''(x) > 0$	tangent lines lie above
$f'(x)$ increasing	trapezoidal approximation of $\int_a^b f(x)\,dx$ will overestimate

$f'(x) < 0$	$f(x)$ decreasing
Left Riemann sum approximation of $\int_a^b f(x)\,dx$ will overestimate	$f''(x) < 0$

Which does not belong?

$\lim_{x \to 0} \frac{\sin x}{x}$ A	B $\lim_{x \to 0} \frac{x^2 + 1}{x + 1}$
C $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$	D $\lim_{x \to 0} \frac{e^x - 1}{x}$

 A	B $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$
C $\lim_{x \to c^-} f'(x) \neq \lim_{x \to c^+} f'(x)$	D $\lim_{x \to c} f(x) = f(c)$

Which of the following? Be prepared to justify your answer.

If $f(x)$ and $g(x)$ are twice - differentiable, which is guaranteed to have at least two points of inflection?

x	-3	-2	-1	0	1	2	3	4	5
$f''(x)$	2	4	0	-3	-1	-2	0	-3	-2
$g''(x)$	2	4	-1	-3	-1	2	4	3	1

Which of the following has a horizontal asymptote at $y = -2$?

$f(x) = \frac{-2x}{x^2 - 4}$ $\quad$ $g(x) = \frac{6 - 4^x}{2^x - 3}$ $\quad$ $h(x) = \frac{4x}{\sqrt{4x^2 + 1}}$

x	2	5	7	9	10
$f(x)$	5	3	4	1	3

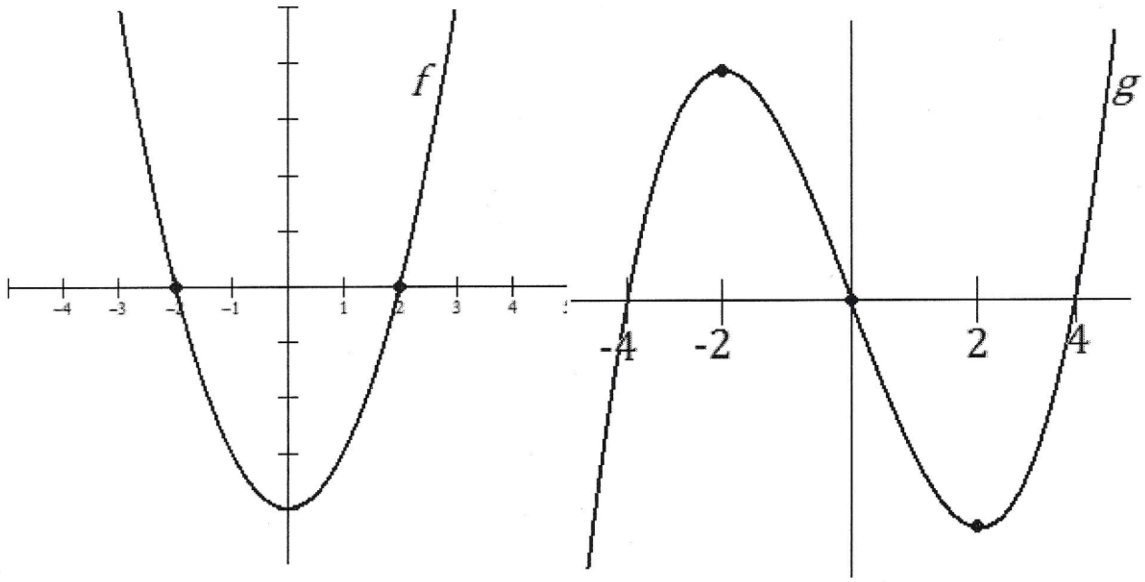
The table above gives selected values of a differentiable function f. Use those values, and the indicated subintervals to find each of the following:

1. Left Riemann sum approximation of the volume of solid formed by rotating $f(x)$ about the x-axis.

2. Trapezoidal approximation of the area of the region enclosed by $f(x)$ and the x-axis.

3. Right Riemann sum approximation of the volume of the solid formed by semicircular cross sections perpendicular to the x-axis and with diameter extending from $f(x)$ to the x-axis.

4. $\int_2^9 f'(x)\,dx$



Put each of the following in order from least to greatest.

1. $f''(0),\ f'(0),\ f(0)$

2. $g''(2),\ g(2),\ \int_{-2}^{2} g(x)dx$

3. $\int_{-2}^{4} g(x)dx,\ \ g'(4),\ g(4)$

4. $\int_{-2}^{-2} g(x)dx,\ \ \int_{-2}^{-4} g(x)dx,\ \ \int_{-2}^{0} g(x)dx$