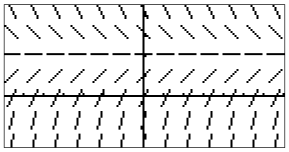
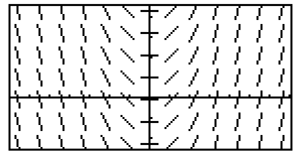
Match the slope fields with their differential equations.

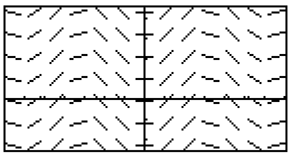
(A)



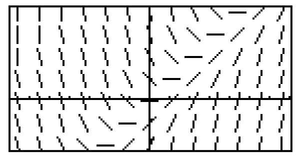
(B)



(C)



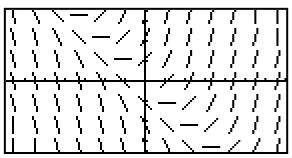
(D)



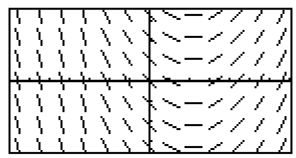
7. $\frac{dy}{dx} = \sin x$ 8. $\frac{dy}{dx} = x - y$ 9. $\frac{dy}{dx} = 2 - y$ 10. $\frac{dy}{dx} = x$

Match the slope fields with their differential equations.

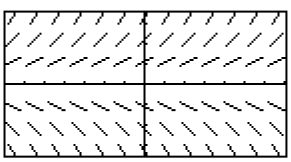
(A)



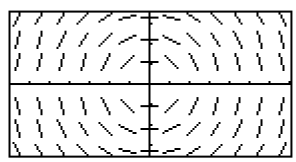
(B)



(C)



(D)



11. $\frac{dy}{dx} = .5x - 1$ 12. $\frac{dy}{dx} = .5y$ 13. $\frac{dy}{dx} = -\frac{x}{y}$ 14. $\frac{dy}{dx} = x + y$

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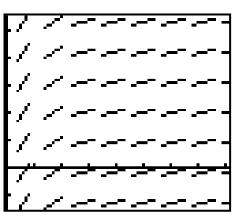
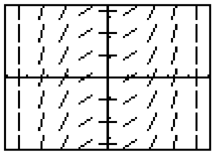
15. (From the AP Calculus Course Description)



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

16.



The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

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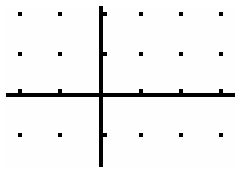
17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(d) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (c). Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

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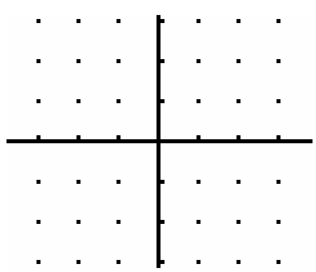
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



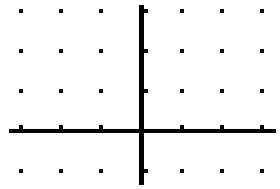
(b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(d) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(e) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

19. Consider the differential equation given by $\frac{dy}{dx} = \frac{2x}{x^2+1}$.



(a) On the axes provided, sketch a slope field for the given differential equation.
(b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(c) Find $\frac{d^2y}{dx^2}$. For what values of x is the graph of the solution $y = f(x)$ concave up? Concave down?

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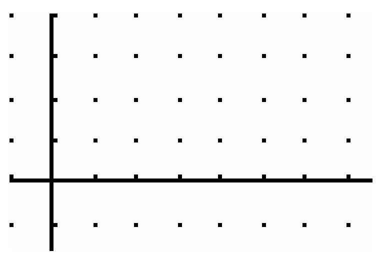
20. Consider the logistic differential equation $\frac{dy}{dt} = \frac{1}{2}y(2-y)$;

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Sketch a solution curve that passes through the point (4, 1) on your slope field.

(c) Show that $y = \frac{2}{1+2e^{-t}}$ satisfies the given differential equation.

(d) Find $\lim_{t\to\infty} y$ by using the solution curve given in part (c).

(e) Find $\frac{d^2y}{dt^2}$. For what values of y, $0 < y < 2$, does the graph of $y = f(t)$ have an inflection point?

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21. (a) On the slope field for $\frac{dP}{dt} = 3P - 3P^2$, sketch three solution curves showing different types of behavior for the population P.

(b) Is there a stable value of the population? If so, what is it?

(c) Describe the meaning of the shape of the solution curves for the population: Where is P increasing? Decreasing? What happens in the long run? Are there any inflection points? Where? What do they mean for the population?

(d) Sketch a graph of $\frac{dP}{dt}$ against P. Where is $\frac{dP}{dt}$ positive? Negative? Zero? Maximum? How do your observations about $\frac{dP}{dt}$ explain the shapes of your solution curves?

(Problem 21 is from Calculus (Third Edition) by Hughes-Hallett, Gleason, et al)