# Rates of Change

## *Average Rate of Change*:

Definition: If a function *f* is continuous on a closed interval $[a,b]$, then the **average rate of change of** $f(x)$ **with respect to *x*** is defined as $\frac{f(b)-f(a)}{b-a}$.

Why Should I Care?: The average rate of change describes the constant rate at which a function would have to change over an interval if the function were to achieve the same vertical displacement over the length of the interval investigated.

Examples:

Bill Dup, who lives in Dallas, takes a trip to visit his brother, Stan, who lives in Pecos. The trip is a 450 mile trip and takes Bill 7.5 hours.

1. How fast is Bill going 4.3 hours into the trip?
2. What is Bill's average velocity? Be sure to include units in your answer.
3. Explain the significance of Bill's average velocity.

A group of St. Mark's seniors have mowed a large field at the West Dallas Community Center to prepare the open field for its conversion to a soccer field. The seniors started mowing at 9 a.m. and stopped mowing at 11:30 a.m. on the same day. The open field measured 60 feet by 150 ft.

1. How fast was Marcello mowing at 10 a.m.?
2. How could you express the average rate at which the field was being mowed by the group of seniors? Be sure to include units in your answer.
3. Explain the significance of this average rate of change.

The Clothing Drive is underway. After 5 school days of collecting the cloths, Mrs. Barta analyzes the progress and reports the following. In those 5 days, 500 shirts were collected, contributing 40 pounds to the weight total. In addition 40 sweaters contributed 75 pounds, and 20 pair of shoes contributed 50 pounds.

1. What was the average rate at which shirts were collected? Include units in your answer.
2. What was the average rate of the shoes when measured in pounds per shoe?
3. If the goal was to collect as many pounds of clothing as possible, what might be the best way to report the 5-day result? How could you use your conclusion to predict the result of an 8 day clothing drive? Include units in your answer.

## *Instantaneous Rate of Change*

Definition: If a function *f* is continuous on a closed interval $[a,b]$, then the **instantaneous rate of change of** $f(x)$ **with respect to *x* at *x* = *c***, for $a < c < b$, is defined as $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$, provided this limit exists.

Why Should I Care?: The instantaneous rate of change describes the rate at which a function is changing at a particular point (at a particular *moment*, in many contexts). It gives us the slope of the best linear model (i.e. the tangent line) for *f* at $x = a$. Also, since very few things in life are constant, a derivative function (i.e. a function that relates the instantaneous rate of change of a function to the point at which that rate is desired) helps mathematicians and scientists evaluate change as it occurs.

Examples:

An object is put outside on a very cold day and the object's temperature, *H*, in C° is a function of time, *t*, in minutes since it was put outside.

1. What does the statement $H(30) = 10$ tell about this context. Be specific and include appropriate units in your answer.
2. What does the statement $H'(30) = -0.5$ tell about this context. Be specific and include appropriate units in your answer.
3. Use the information above to estimate the temperature of the object 2 hours after it is placed outdoors. Is this likely to be a good estimate? Explain.

If an object is tossed vertically upward, its directed distance from the point of release is given by $s = v_o t - \frac{g}{2} t^2$ where $v_o$ represents the initial velocity and $g$ is a constant describing the acceleration due to gravity. Stan Dup is frustrated after a Calculus test, and, after leaving the math building, tosses his text vertically into the air (in hopes it never returns to the ground). He releases the text from a point 2 meters above the ground with an initial velocity of 10 $\frac{m}{\text{sec}}$. Stan knows that $g = 9.8 \frac{m/\text{sec}}{\text{sec}}$.

1. How far above the ground is the textbook after 2 seconds?
2. How fast is it going at the end of 2 seconds?
3. What is the highest point above ground that the textbook will reach?
4. What is the average velocity of the textbook between $t = 1$ and $t = 2$?