Are we asking the right questions? CORRECTED VERSION

The twice-differentiable function $f(x)$ has values as given in the table below. $f'(x)$ is increasing on the interval $-3 \le x \le 2$.

x	-3	-2	1	2	5
$f(x)$	2	3	5	8	2
$f'(x)$	0	0.3	0.8	4	-3

1. Is there guaranteed to be a value *c* in the interval $2 < x < 5$ such that $f(c) = 6$? Justify your answer.

2. Is there guaranteed to be a critical point in the interval $-3 < x < 5$? Justify your answer.

3. Over what interval is there guaranteed to be a value *c* such that $f'(c) = 1$?

4. $\frac{f''(5)-f''(2)}{5-2} = -1$, yet there is no value *c* in the interval $2 < x < 5$ such that $f'''(c) = -1$. What does this tell you about $f''(x)$?

5. Write the equation of the line tangent to $y = f(x)$ at $x = 1$ and use it to approximate $f(1.2)$. Is your approximation less than or greater than the actual value of $f(1.2)$? Explain.

6. If $g(x) = x^3 f(x)$, find $g'(-2)$.

7. If $h(x) = f(x^3)$, then is h increasing at an increasing rate or increasing at a decreasing rate at $x = 1$? Explain.

8. Find the value of $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ or state that it does not exist.

9. Find the value of $\lim_{x\to 1} \frac{f(x)-5}{x^2-1}$ or state that it does not exist.

10. Find the value of $\lim_{n\to\infty} \sum_{k=1}^{n} f'(-2 + \frac{7k}{n}) \cdot \frac{7}{n}$ or state that it does not exist.

11. Approximate $\int_{-3}^{2} f(x)dx$ using 3 trapezoids with the sub-intervals indicated by the table. Is this approximation less than or greater than the actual value of $\int_{-3}^{2} f(x)dx$. Explain.

12. Find the value of $\int_{-2}^{5} f(x)f'(x)dx$.

13. Find the value of $\int_{0}^{2} (f''(2x+1) + 3)dx$.

14. Find the value of $\frac{d}{dx}\int_{-3}^{2x^2} f(t)dt$ at $x = 1$.

15.