A problem for our times:

The rate at which people enter a large store is modeled by the differentiable function $E(t)$ and the rate at which they leave the same store is modeled by the differentiable function $L(t)$, as given below.

$$E(t) = 20 + \frac{100}{1+\ln(t+1)} \qquad\qquad L(t) = 120\sin\left(\frac{t^2}{47}\right)$$

Both $E(t)$ and $L(t)$ are measured in people per hour and t is measured in hours since the store opened at $t = 0$. There are 50 employees already in the store when it opens. The store is open over the time interval $0 \le t \le 12$.

a) Using correct units, explain the meaning of $E'(6)$ in the context of the problem.

b) Using correct units, explain the meaning of $\frac{1}{4}\int_3^7 E(t)dt$ in the context of the problem.

c) At $t = 10$, is the number of people in the building increasing or decreasing? Justify your answer.

d) At $t = 10$, is the rate of change of the number of people in the building increasing or decreasing? Justify your answer.

e) At what time are the maximum number of people in the store? How many people are in the store at that time? Justify your answer.

f) During what time interval do people leave the store at a rate greater than 80 people per hour? What is the average rate that people leave the store during that time interval?

g) Write an expression that could be evaluated to determine the average rate of change in the rate that people leave the building between $t = 6$ hours to $t = 12$ hours.

h) When $t = 12$ hours, the entrance is locked and no more people are allowed to enter the building. Write an equation involving an integral that could be solved to predict the time at which no one will remain in the building.